



**NANYANG
TECHNOLOGICAL
UNIVERSITY**

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence



MA4822

Measurement and Sensing Systems

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Outline

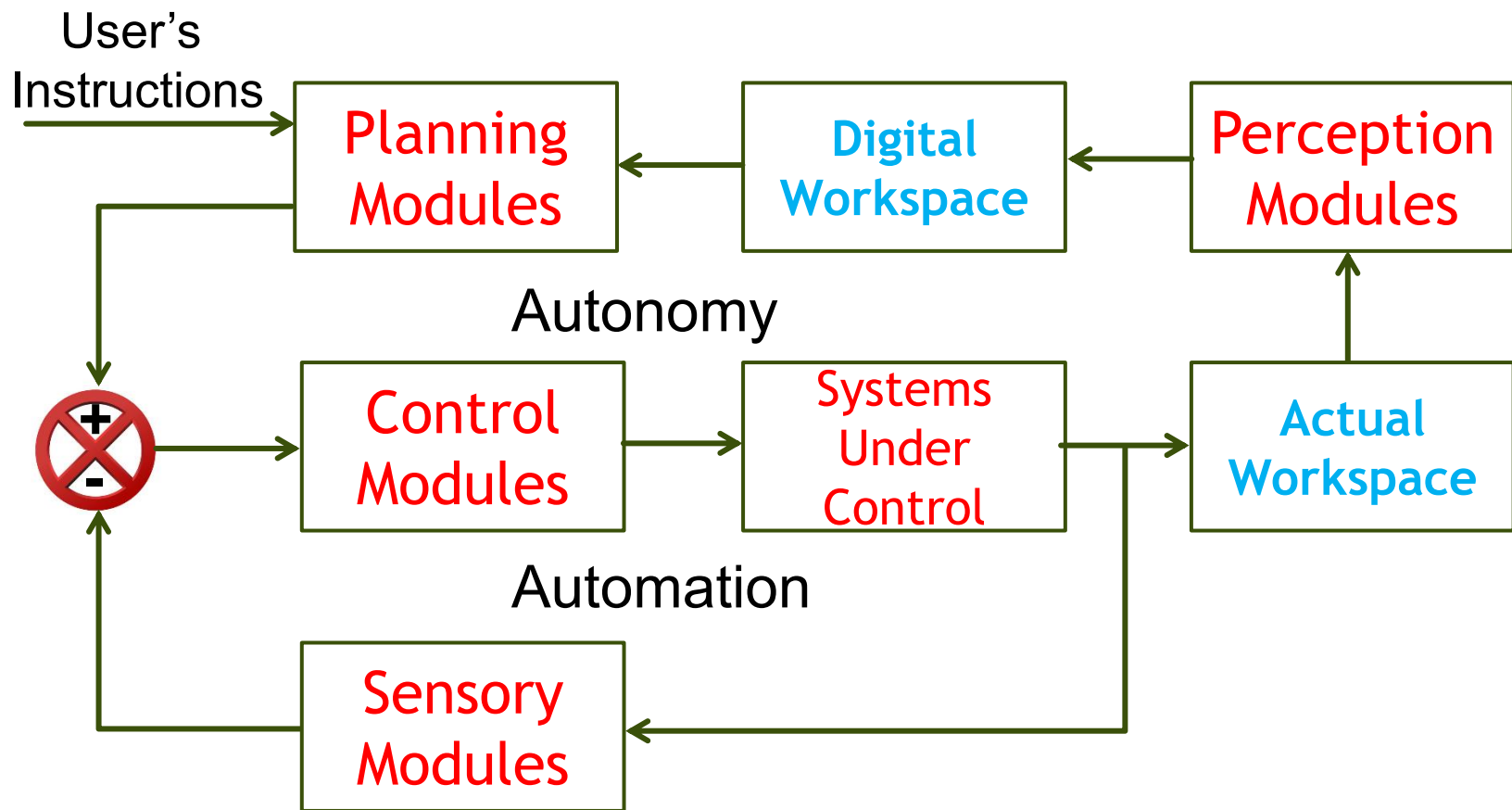
- ▶ Module 1: Foundation of AI Sensors
- ▶ Module 2: Sensors for Electrical Systems
- ▶ Module 3: Sensors for Mechanical Systems
- ▶ Module 4: Sensors for All Environments
- ▶ Module 5: Sensors for All Industries

Remember your mission as MAE undergraduates ...

- ▶ You are here to grow your knowledge and skills so as to be able to design machines with **controllable behaviors** and hopefully in some **intelligent ways**.

How to fulfill your mission?

- ▶ To apply learnt knowledge and skills into the implementation of the following universal blueprint underlying all the intelligent machines or systems.



Why to study this course?



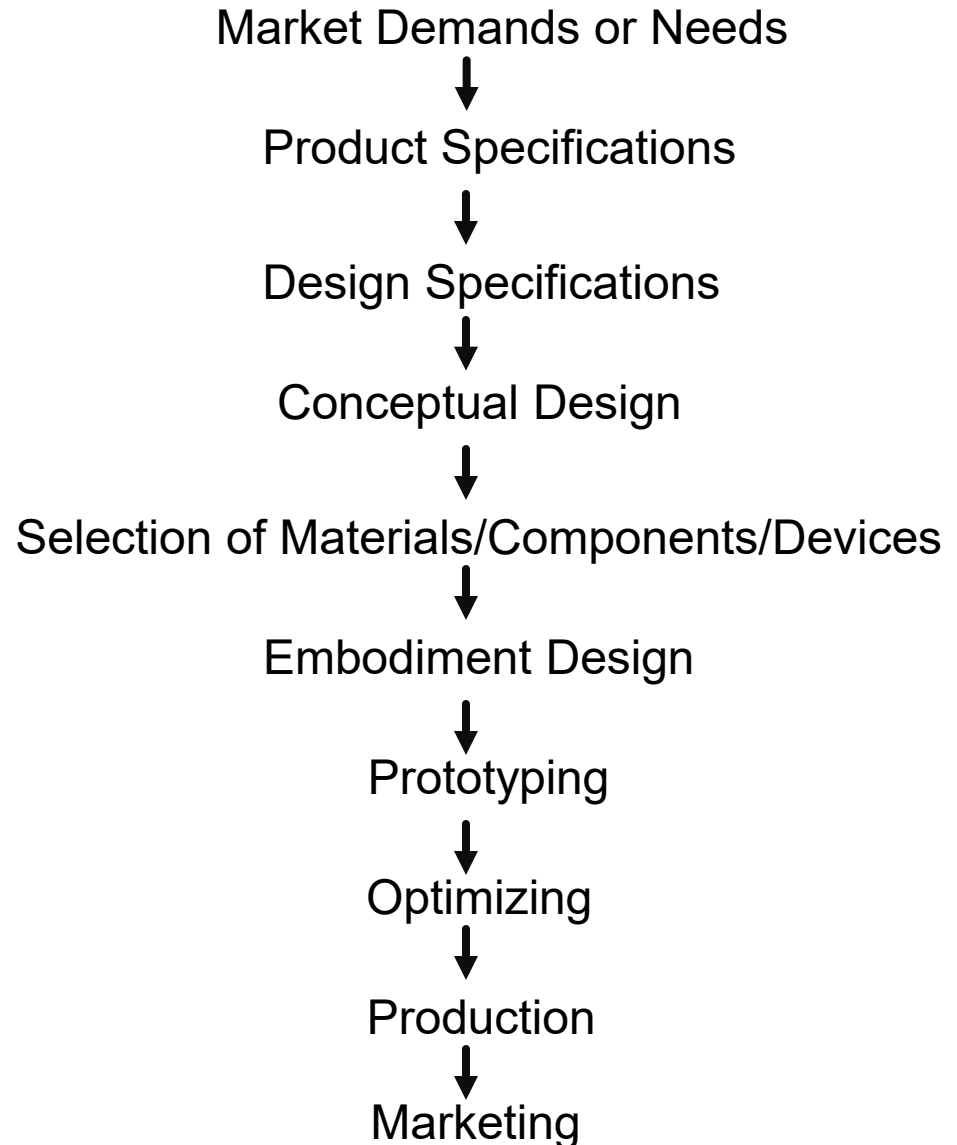
We are living inside an ocean of signals

(Learning, Teaching) <o> (Research, Innovation) <o> (Leadership, Service)

How to study this course?

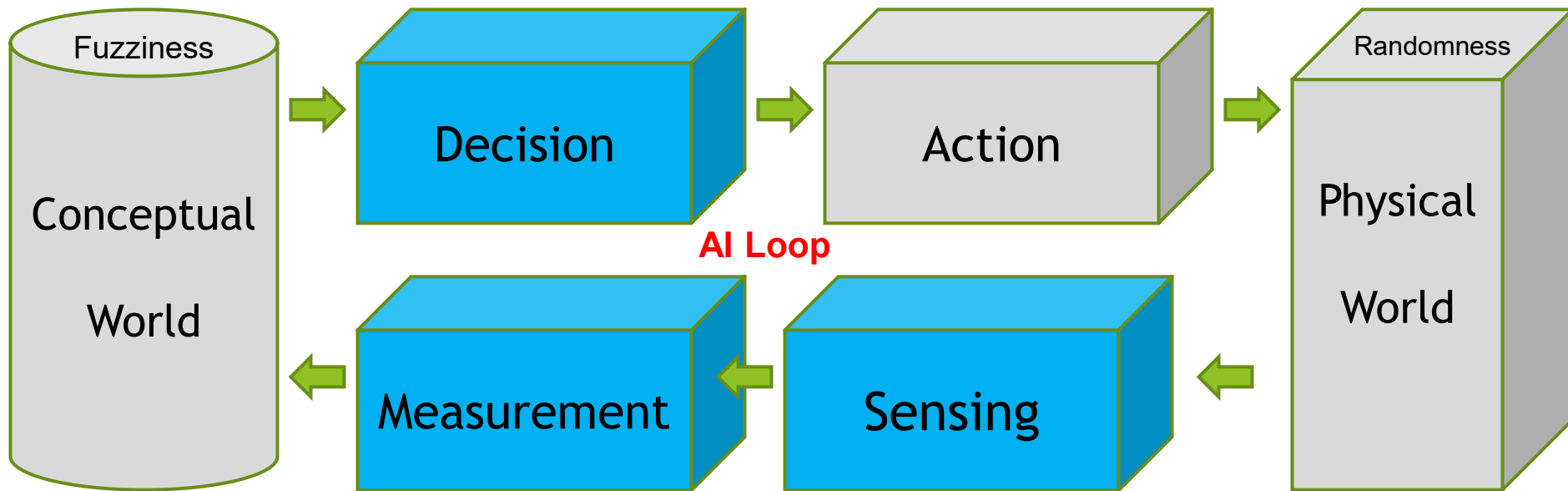
- ▶ To put yourselves into the mindset of designers of networked sensors as products:
 - ▶ Who are the users?
 - ▶ What are the needs of users?
 - ▶ What are your Internet of Sensors, which could meet the needs of your users or buyers?
 - ▶ What are the solutions behind the design of your Internet of Sensors?

Practice with MATLAB



What are you going to study in this course?

- Module 1: Foundation of AI Sensors
- Basics of Physical World
 - Randomness of Physical World
 - Basics of Conceptual Worlds
 - Fuzziness of Conceptual Worlds



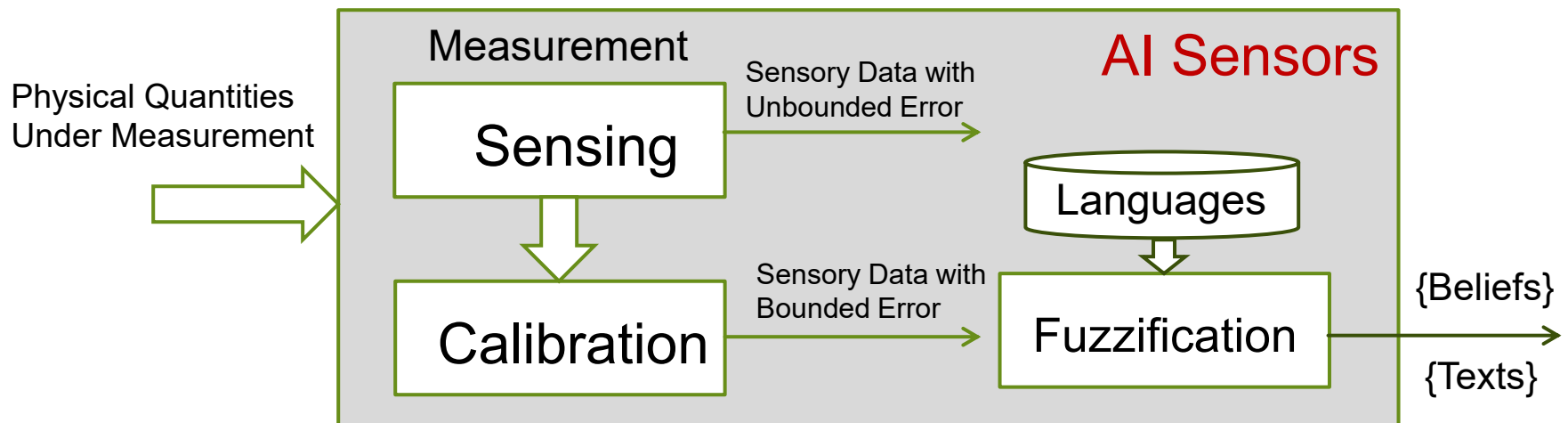
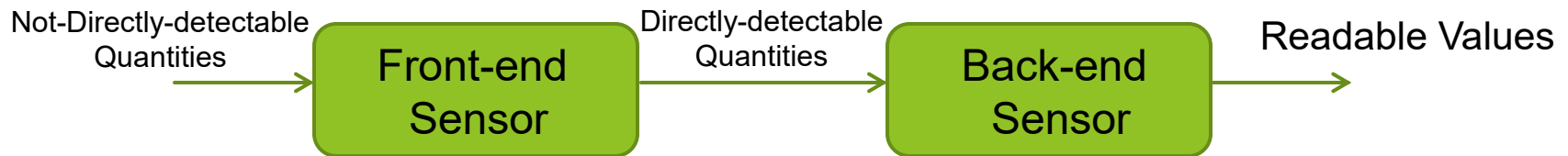
- Module 2: For Electrical Systems
- Measurement of Voltage
 - Measurement of Current
 - Measurement of Resistance
 - Measurement of Capacitance
 - Measurement of Inductance

- Module 3: For Mechanical Systems
- Measurement of Position
 - Measurement of Velocity
 - Measurement of Acceleration
 - Measurement of Force
 - Measurement of Torque

- Module 4: For All Environments
- Measurement of Pressure
 - Measurement of Temperature
 - Measurement of Humidity
 - Measurement of Vibration
 - Measurement of Air Quality

- Module 5: For All Industries
- Measurement of Fluid Level
 - Measurement of Flow Rate
 - Measurement of Sound/Voice
 - Measurement of Photometry
 - Measurement of Geometry

How to apply?



Today's Lectures ...

- ▶ **Module 1: Foundation of AI Sensors**
- ▶ Module 2: Sensors for Electrical Systems
- ▶ Module 3: Sensors for Mechanical Systems
- ▶ Module 4: Sensors for All Environments
- ▶ Module 5: Sensors for All Industries



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Module 1

MA4822

Foundation of AI Sensors

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Outline of Module 1

▶ Lecture 1:

▶ Basics of Physical World

▶ Lecture 2:

▶ Randomness of Physical World

▶ Lecture 3:

▶ Basics of Conceptual Worlds

▶ Lecture 4:

▶ Fuzziness of Conceptual Worlds



Outline of Module 1

- ▶ Lecture 1:
 - ▶ Basics of Physical World
- ▶ Lecture 2:
 - ▶ Randomness of Physical World
- ▶ Lecture 3:
 - ▶ Basics of Conceptual Worlds
- ▶ Lecture 4:
 - ▶ Fuzziness of Conceptual Worlds





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Module 1 Lecture 1

MA4822

Basics of Physical World

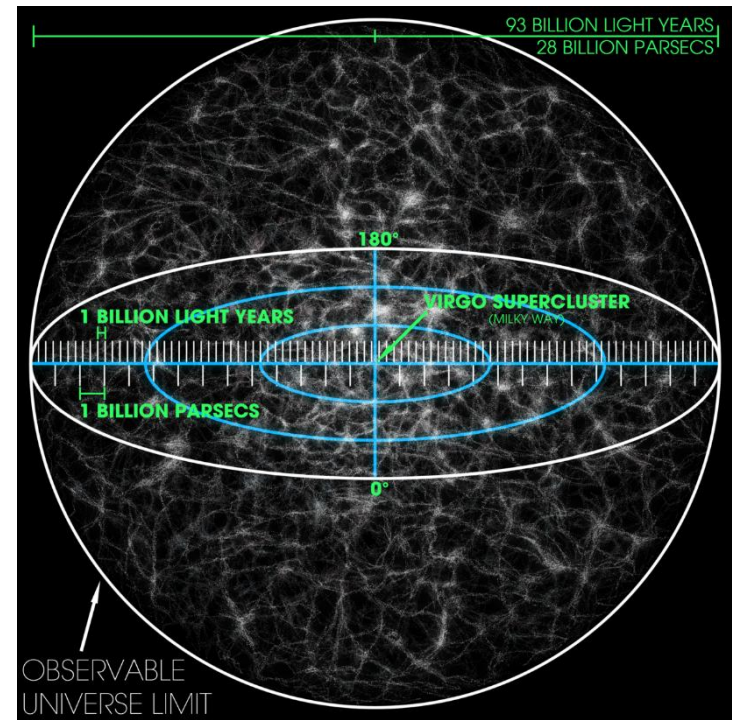
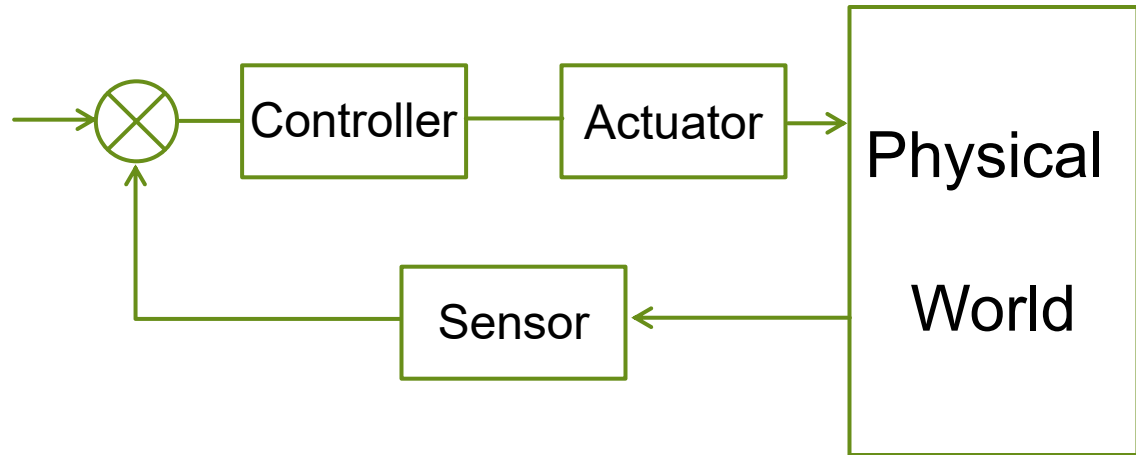
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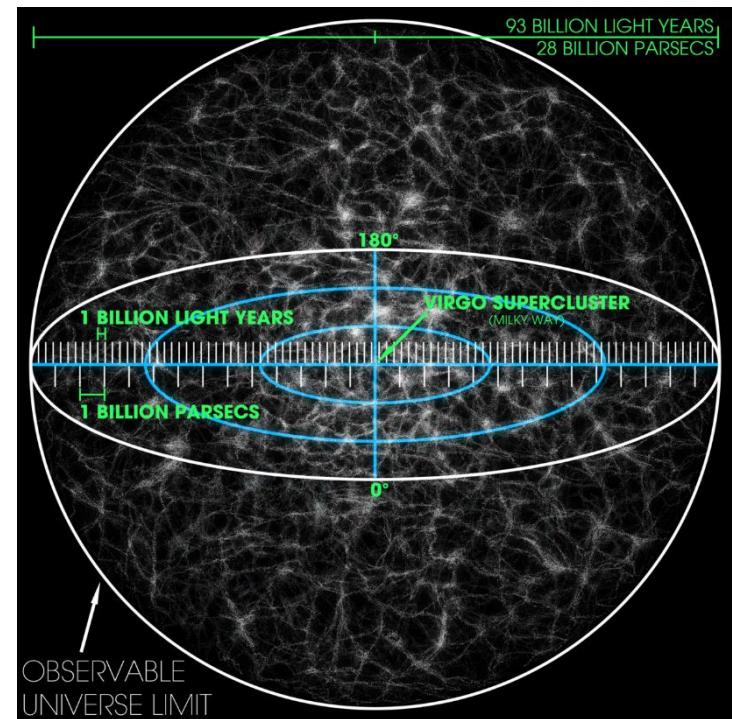
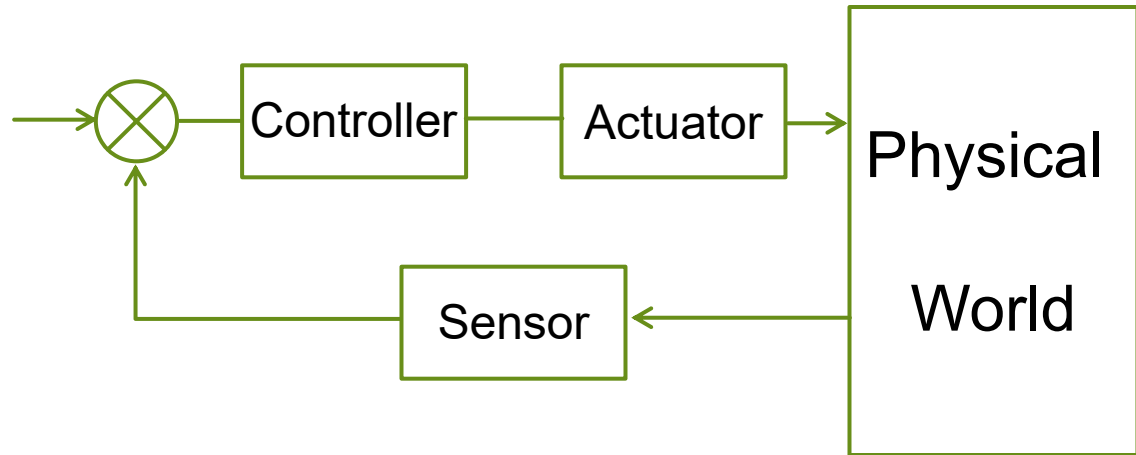
Outline

- ▶ Physical Systems
- ▶ Physical Entities
- ▶ Properties of Physical Entities
- ▶ Constraint of Physical Entities
- ▶ Physical Quantities
- ▶ Measurement by Comparison
- ▶ Measurement by Sensing



Outline

- ▶ Physical Systems
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How big is the universe?



Let's talk about
size

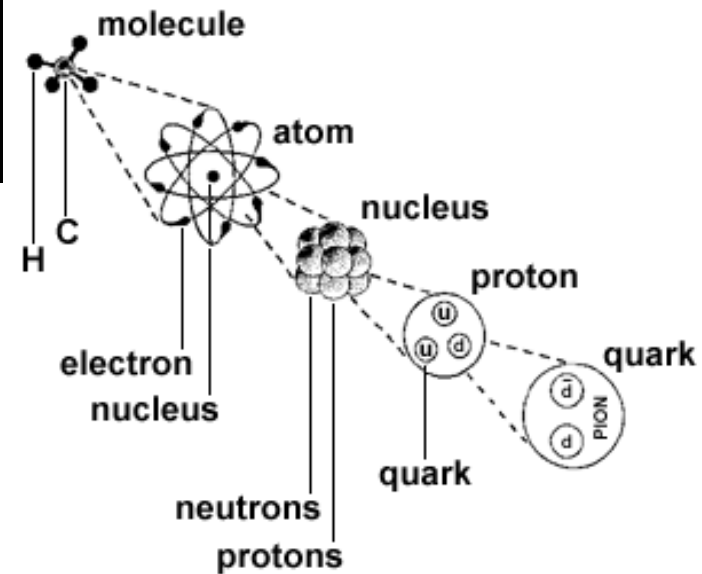
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Dimension of the Universe (1)



- Very Large
- Very Small



Dimension of the Universe (2)

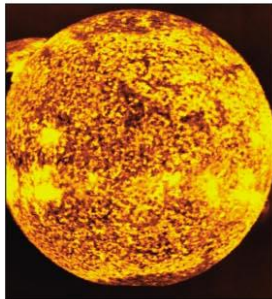
10^{21} m
A galaxy



10^{-5} m
Red blood cells

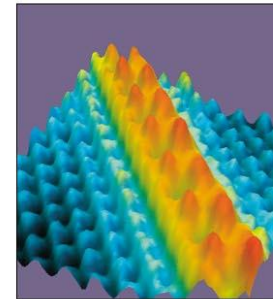


10^9 m
Our sun

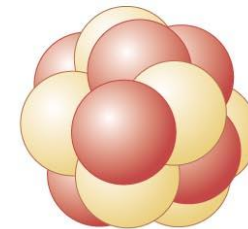


10^0 m
You

10^{-9} m
Individual atoms

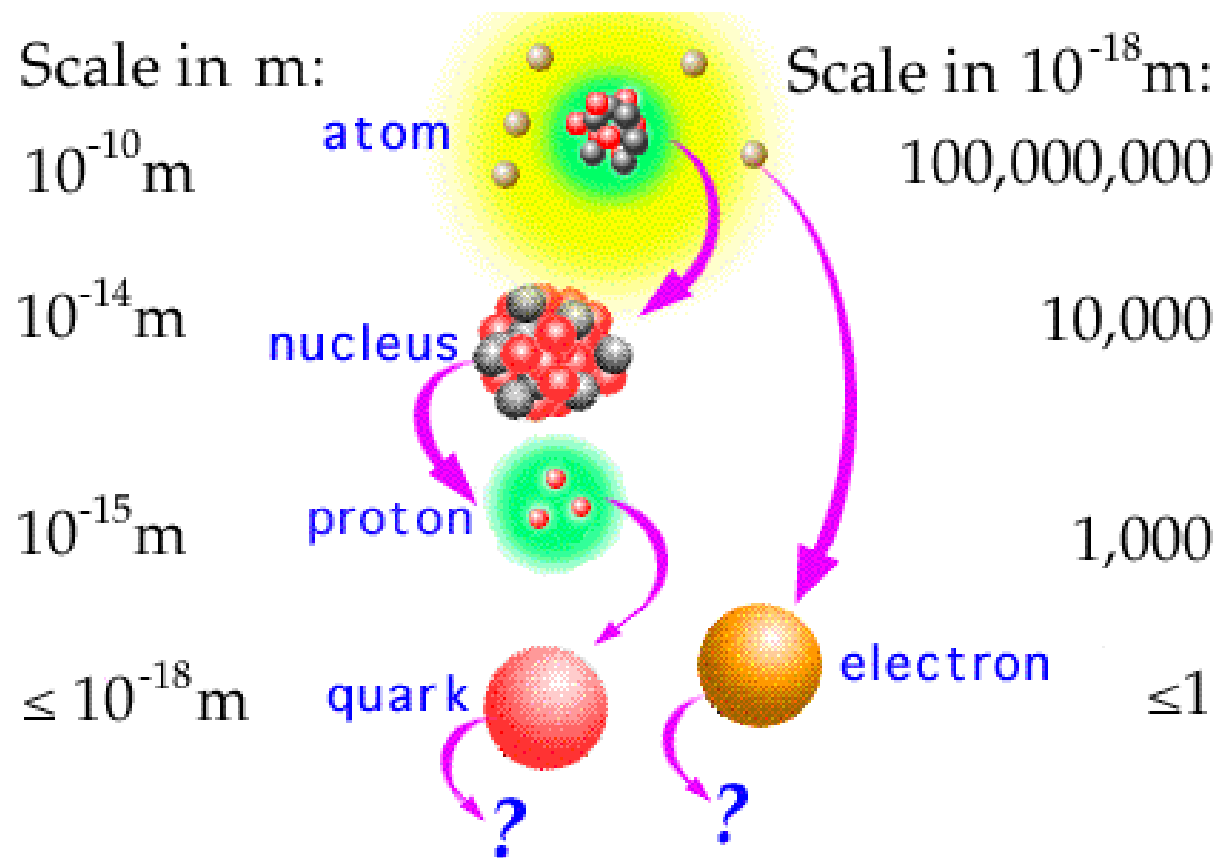


10^7 m
The earth



10^{-14} m
An atomic nucleus

Dimension of the Universe (3)



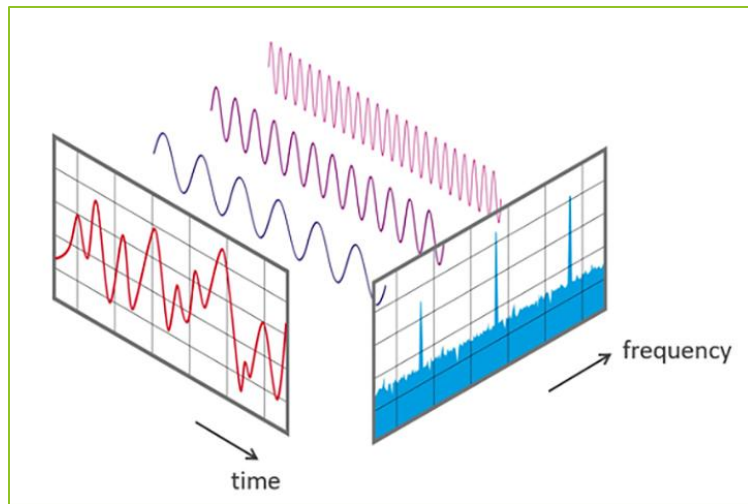
What is the universe?

“One Universe, Two Worlds”, - Xie Ming



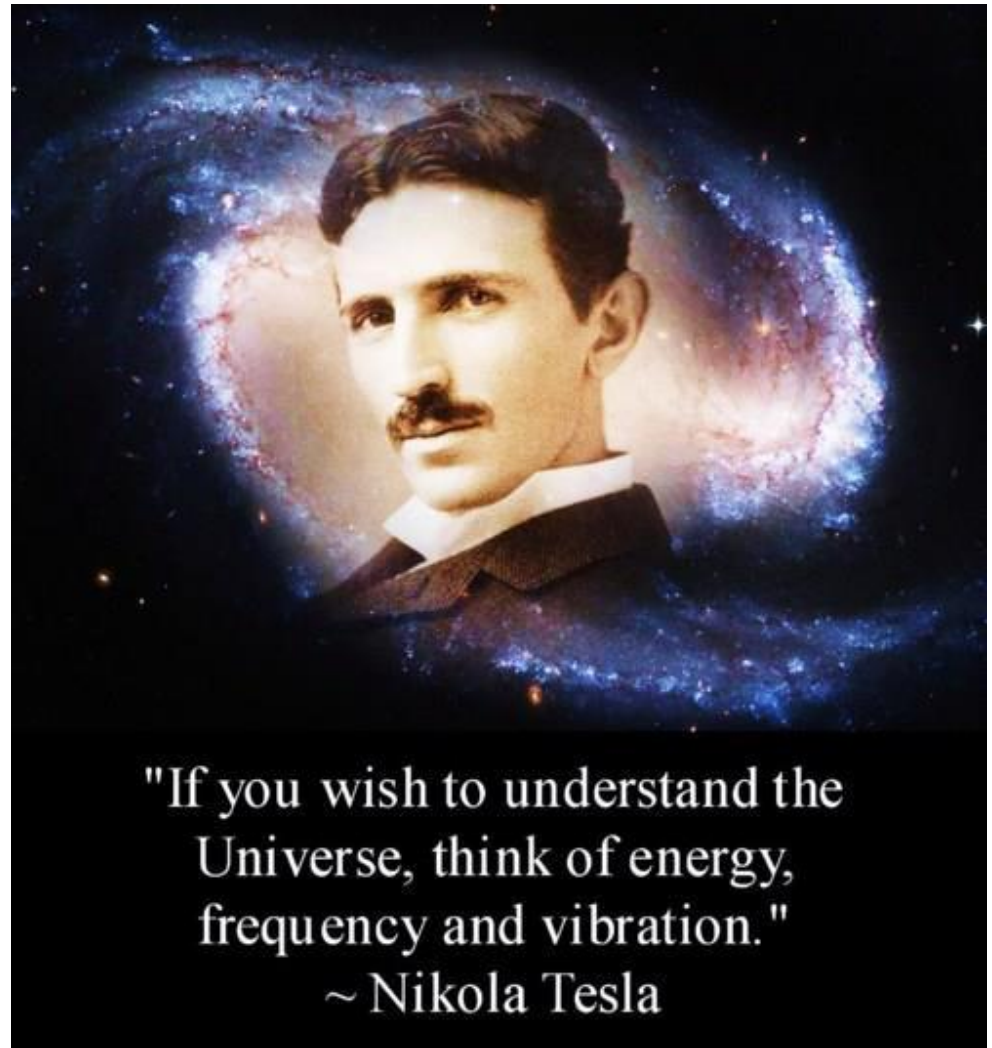
Very Important Discovery ...

Nikola Tesla (1856 - 1943)



Fourier Transform

Joseph Fourier (1768 - 1830)



Very Important Discovery (continued)

- ▶ If you want to understand the organization of the Universe, think in terms of:

- ▶ Systems
- ▶ Devices
- ▶ Materials

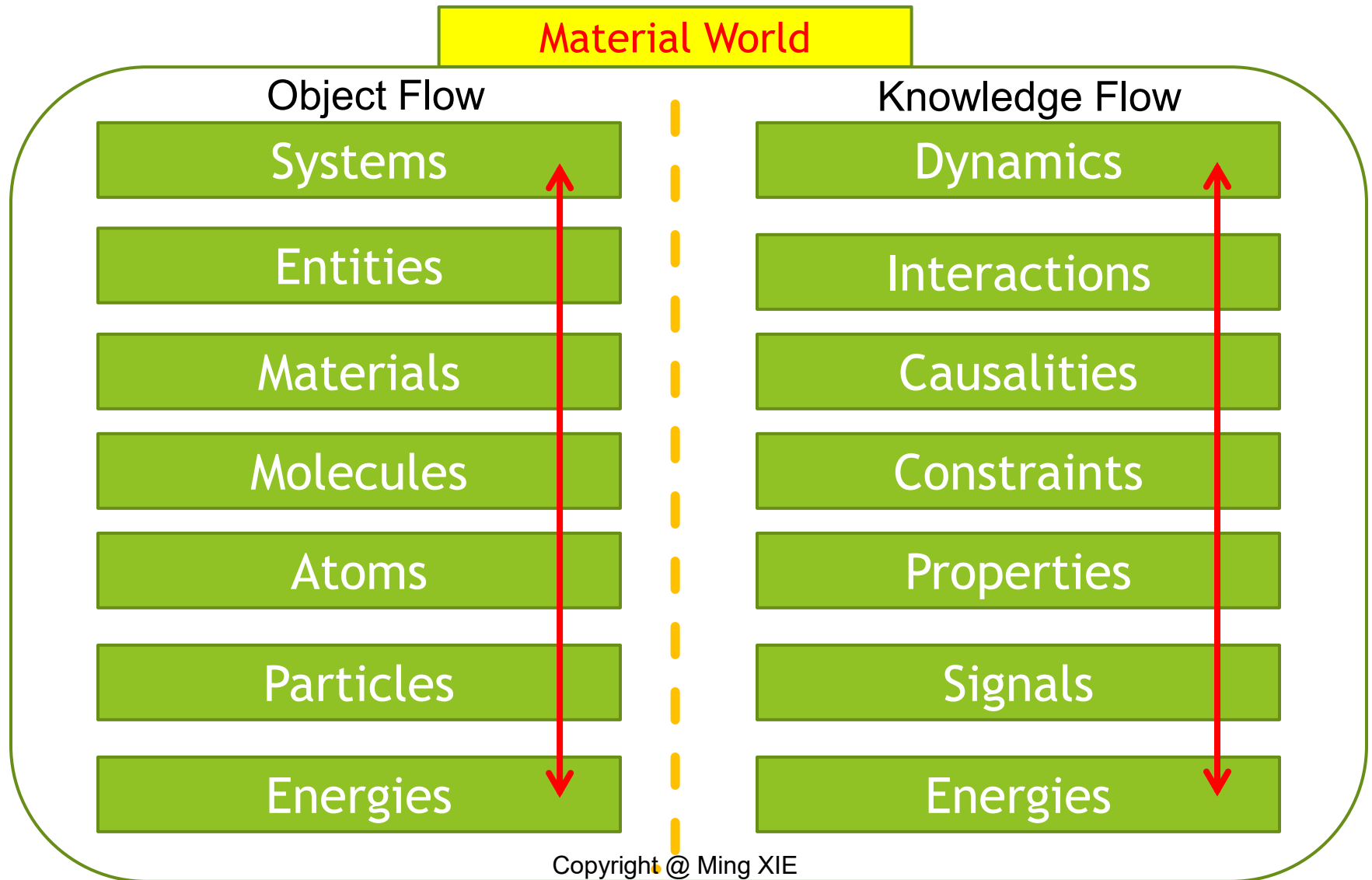


- ▶ If you want to understand the meanings of the Universe, think in terms of:

- ▶ Space
- ▶ Vector
- ▶ Time

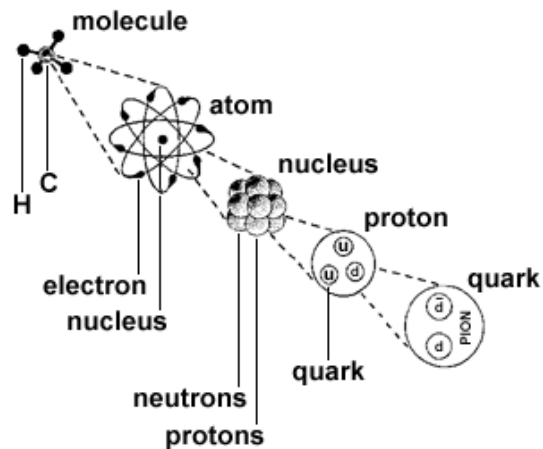


What is the universe? (Material World + Spiritual World)



How is the universe organized?

- ▶ System Levels
- ▶ Entity Levels
- ▶ Material Levels



What are physical systems?

- ▶ Physical entities in the physical world could act and interact together.
- ▶ All the physical entities which act and interact together for the purpose of achieving common goals are forming specific groups which are called as Systems.

What are typical responses from a system?

- ▶ A system has:
 - ▶ Steady-state responses
 - ▶ Transient responses

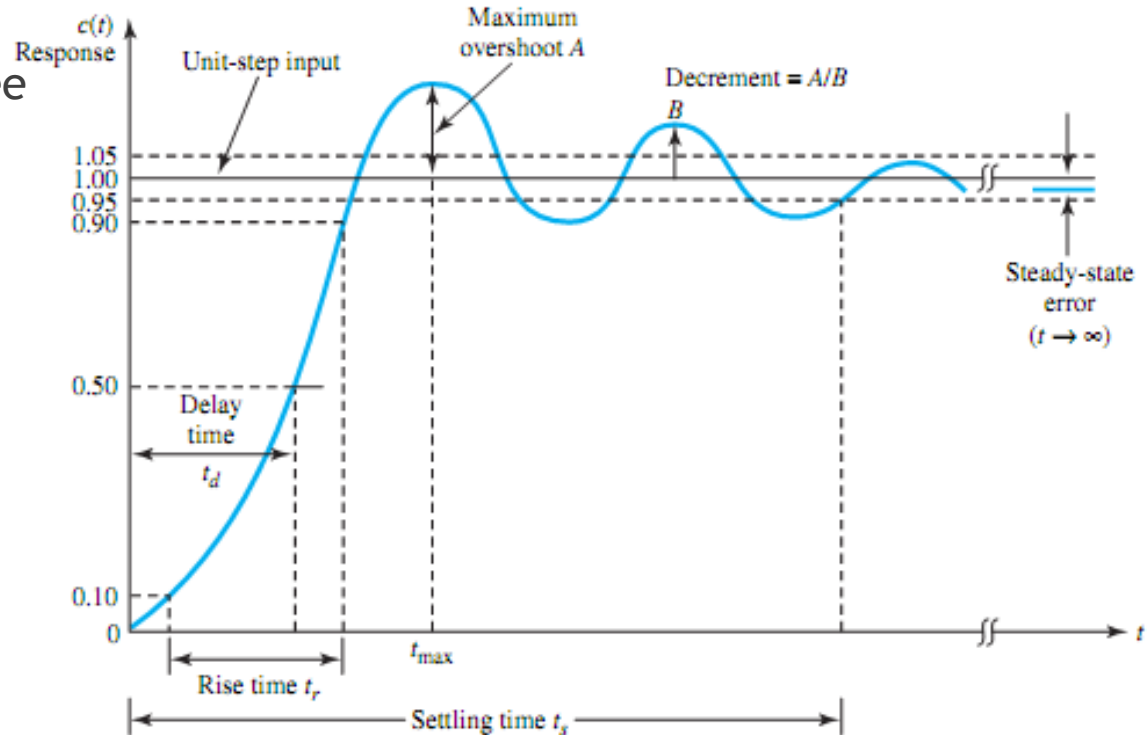
1. One System
2. Two Responses
3. Three KPIs
4. Four Design Methods

- ▶ A system is evaluated by three indicators of responses:

- ▶ Stability
- ▶ Response time
- ▶ Response accuracy

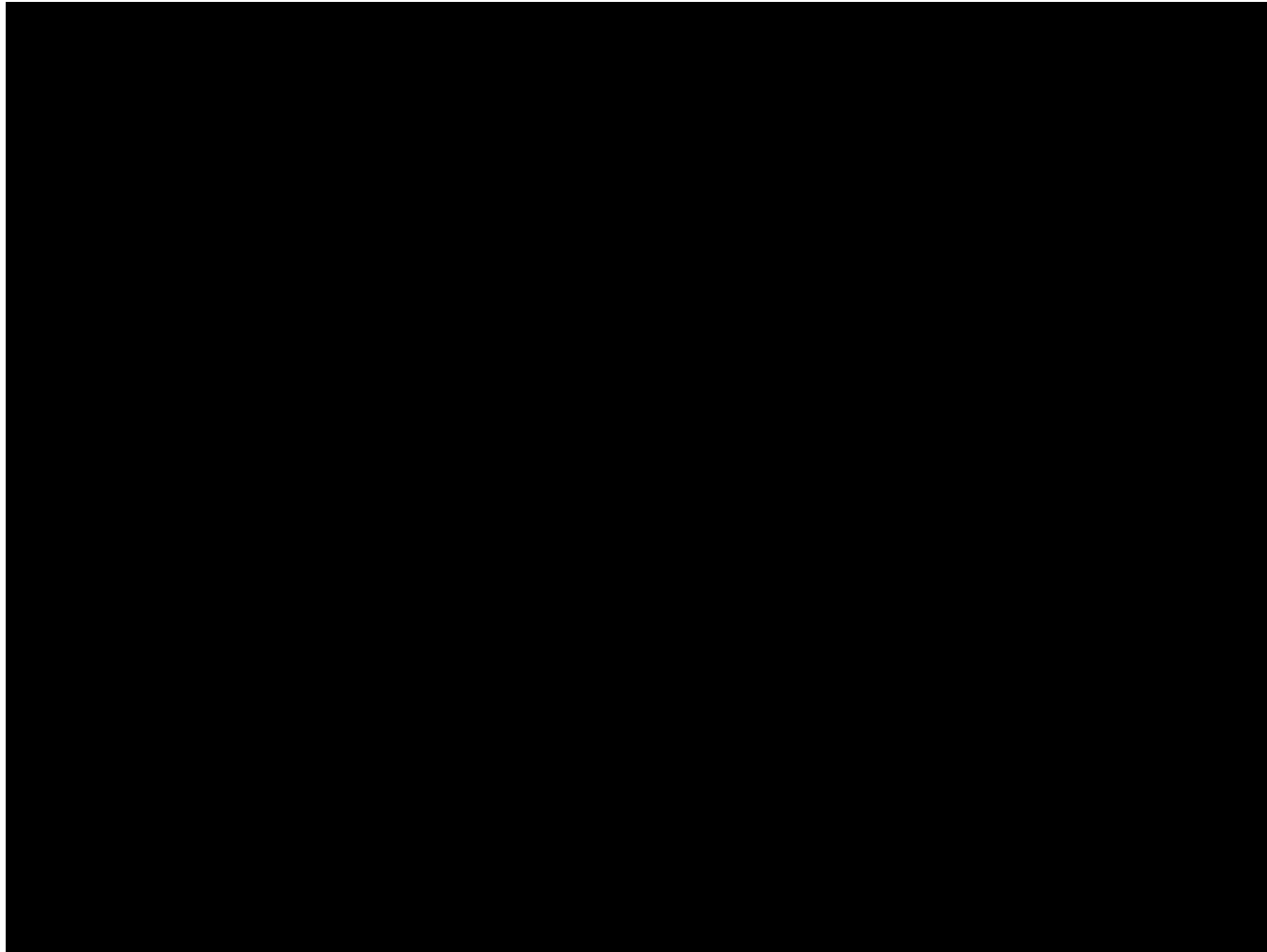
- ▶ Hence, the best systems on Earth are the **static systems**.

- ▶ The ideal systems will be the **closed-loop control systems**

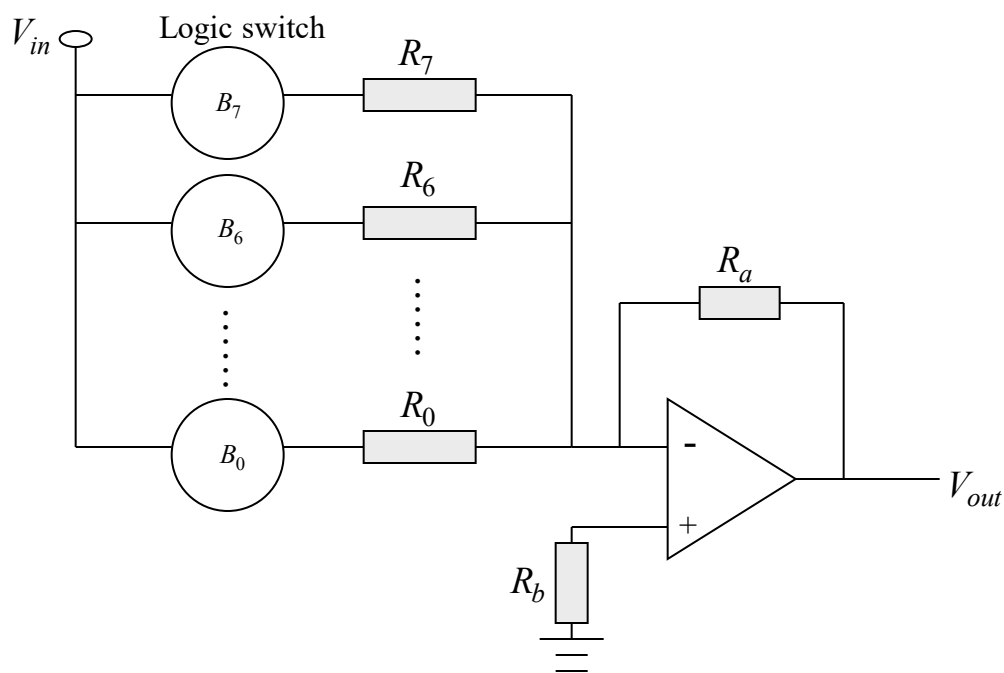


Typical unit-step response of a control system.

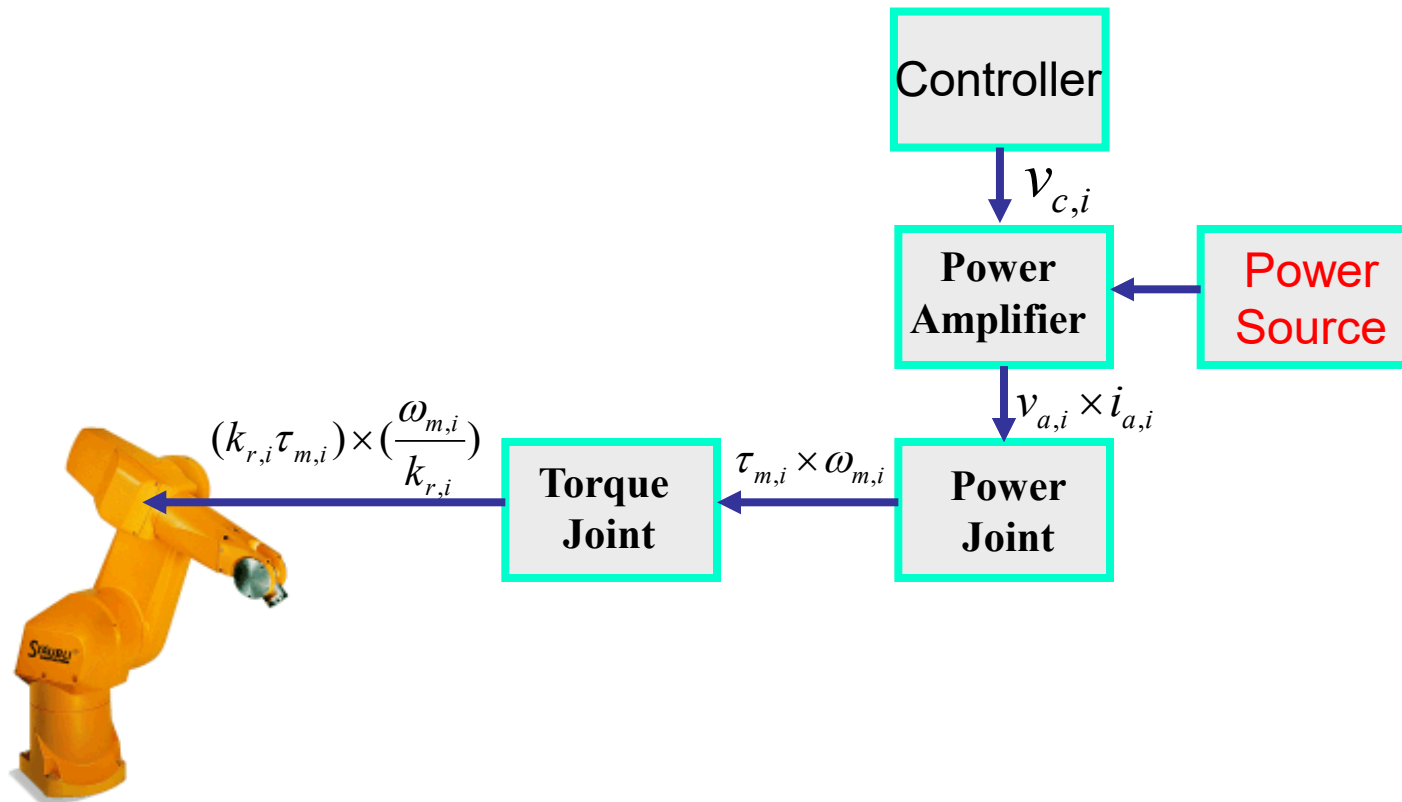
Example of Human-Made Systems



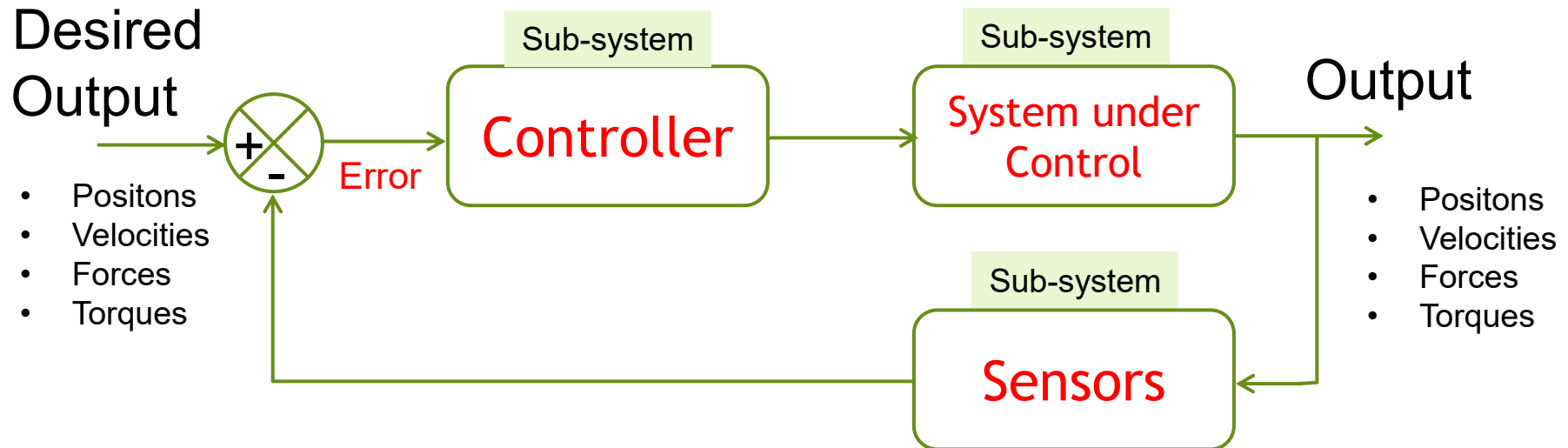
Example of Dynamic Systems without Control



Example of Dynamic Systems with Control

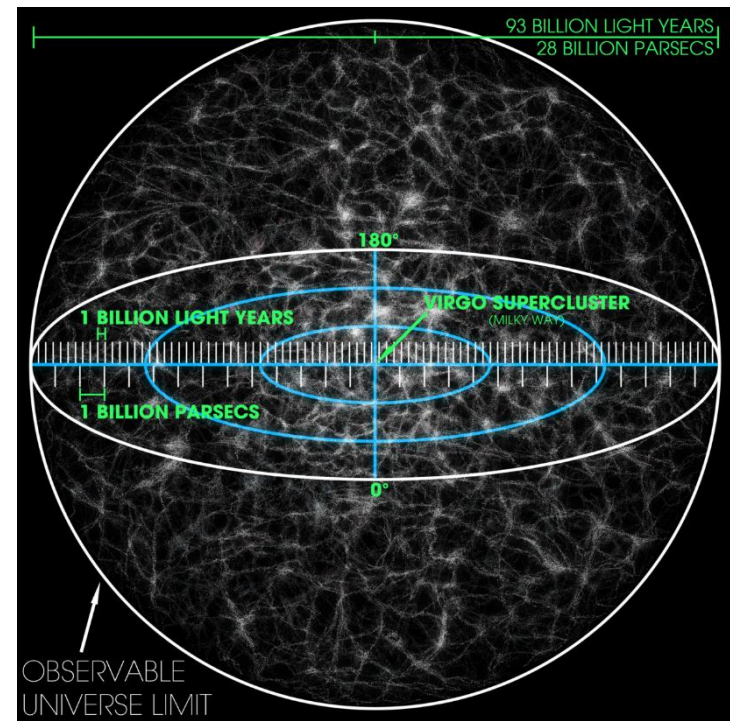
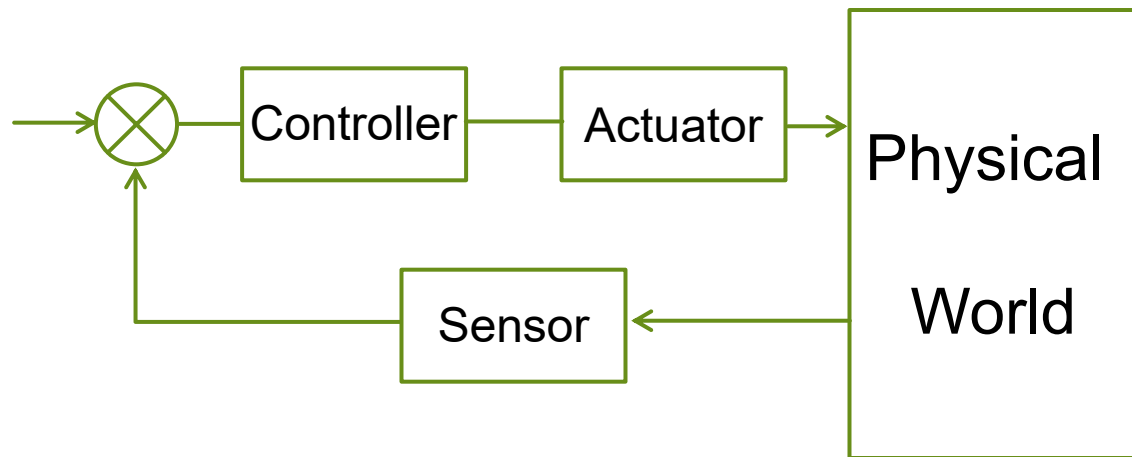


Blueprint of Dynamic Systems with Control



Outline

- ▶ Physical Systems
- ▶ Physical Entities
- ▶ Properties of Physical Entities
- ▶ Constraint of Physical Entities
- ▶ Physical Quantities
- ▶ Measurement by Comparison
- ▶ Measurement by Sensing



What are physical entities?

- ▶ Physical entities refers to all matters which exist in the universe.
- ▶ The physical entities form the physical world.
- ▶ The physical entities on Earth are visible to human eyes or machines' sensors.

Examples of Physical Entities on Earth ...



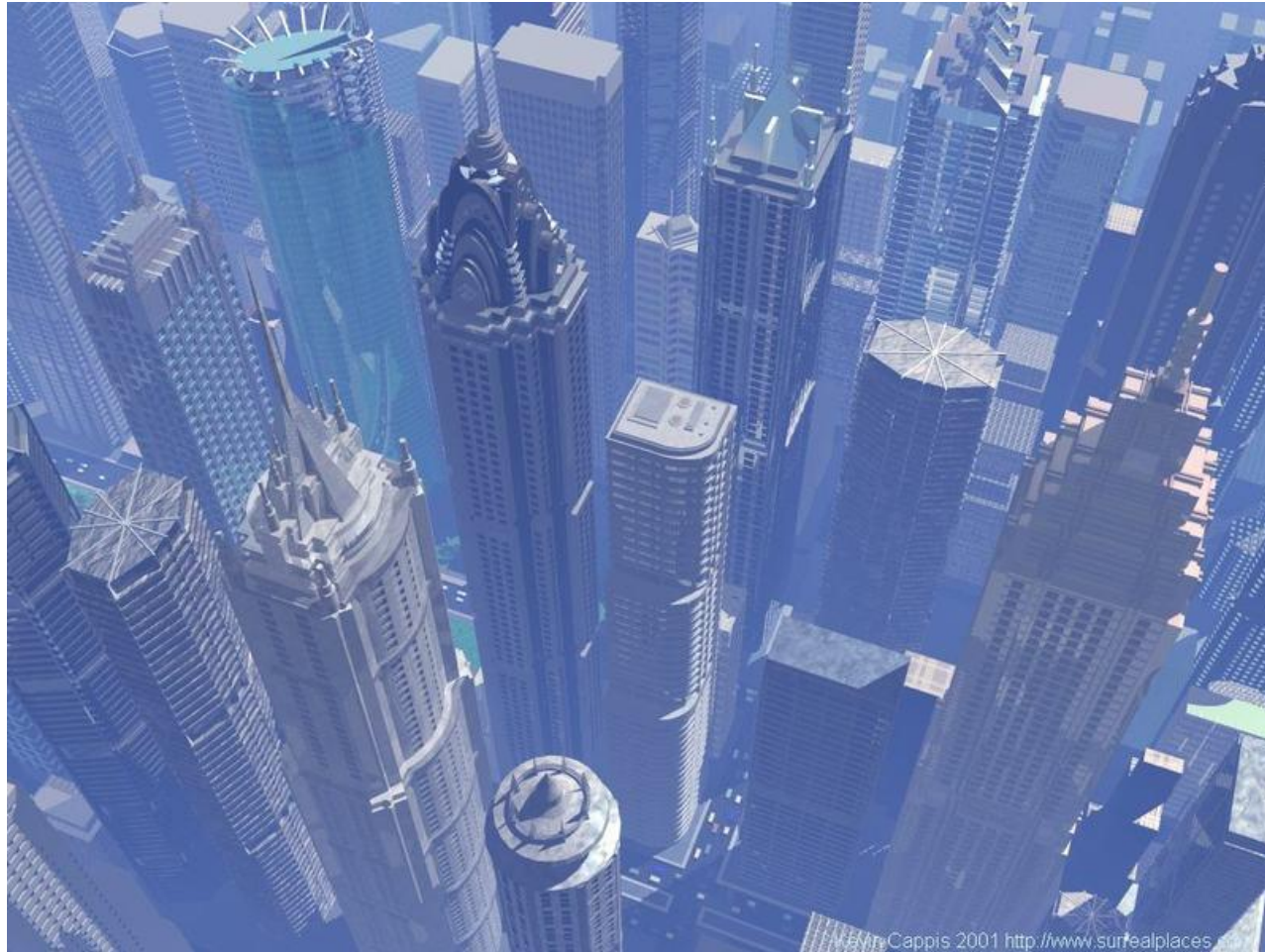
Plants



Rivers

Examples of Physical Entities on Earth (continued)

...



Buildings

Examples of Physical Entities on Earth (continued)

...



Bridges

Examples of Physical Entities on Earth (continued)

...



Roads

Examples of Physical Entities on Earth (continued)

...



Cars and Streets

Examples of Physical Entities on Earth (continued)

...



Ships and Boats

Examples of Physical Entities on Earth (continued)

...



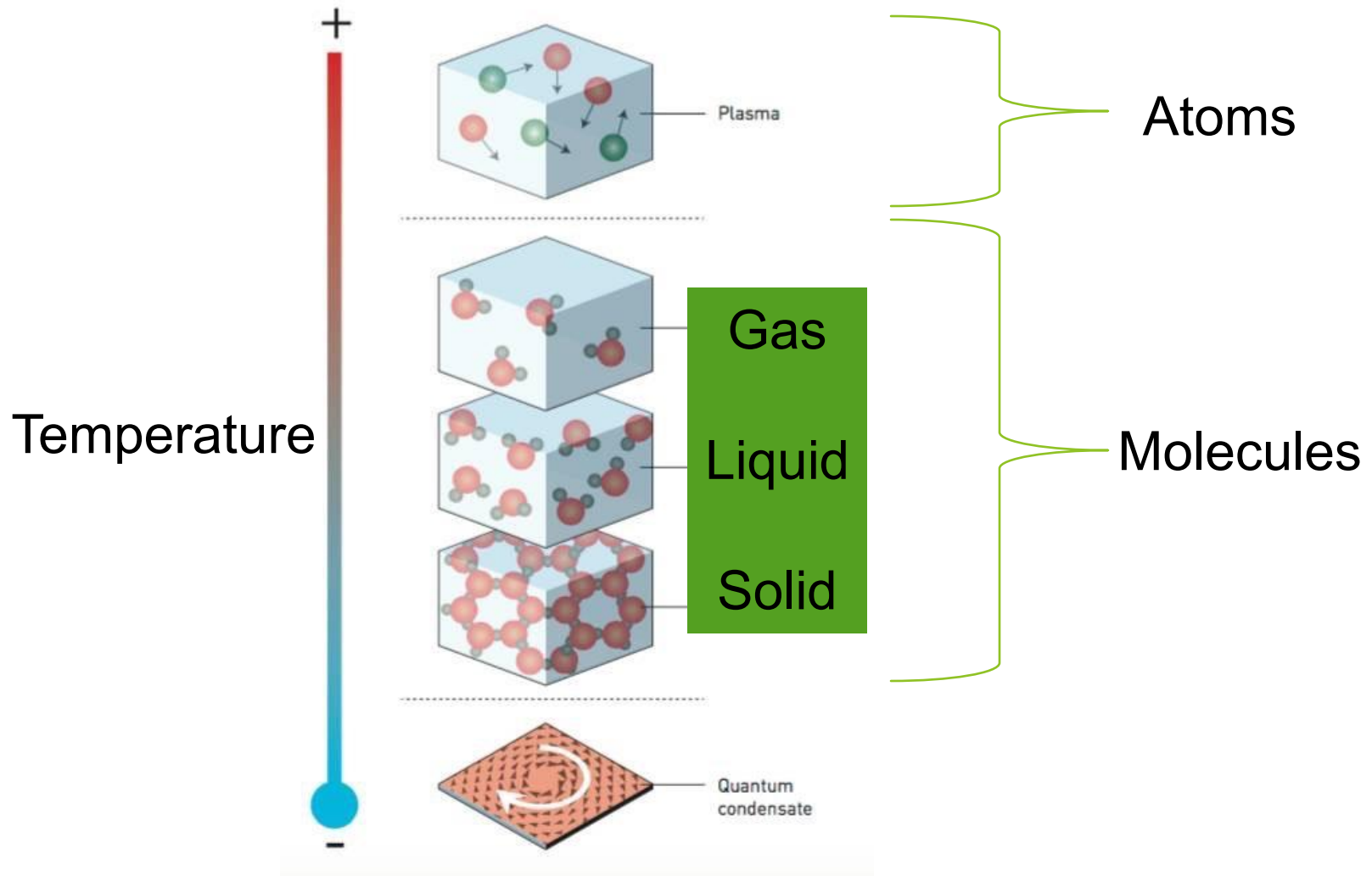
Airplanes

Examples of Physical Entities on Earth (continued) ...

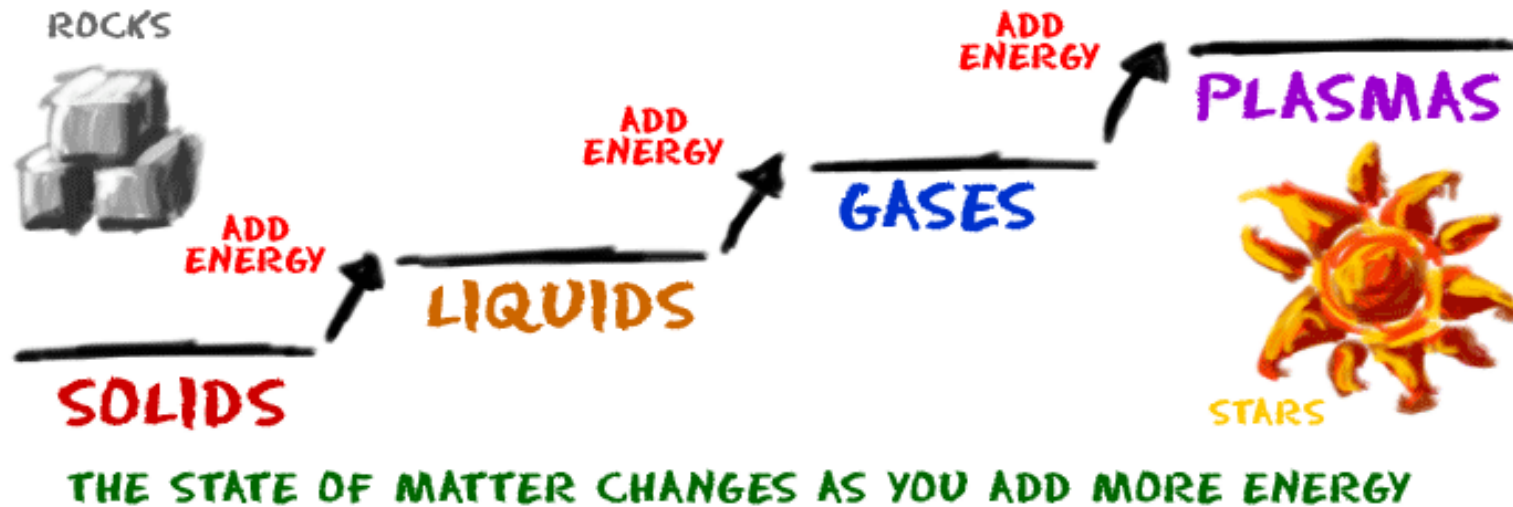


Tanks

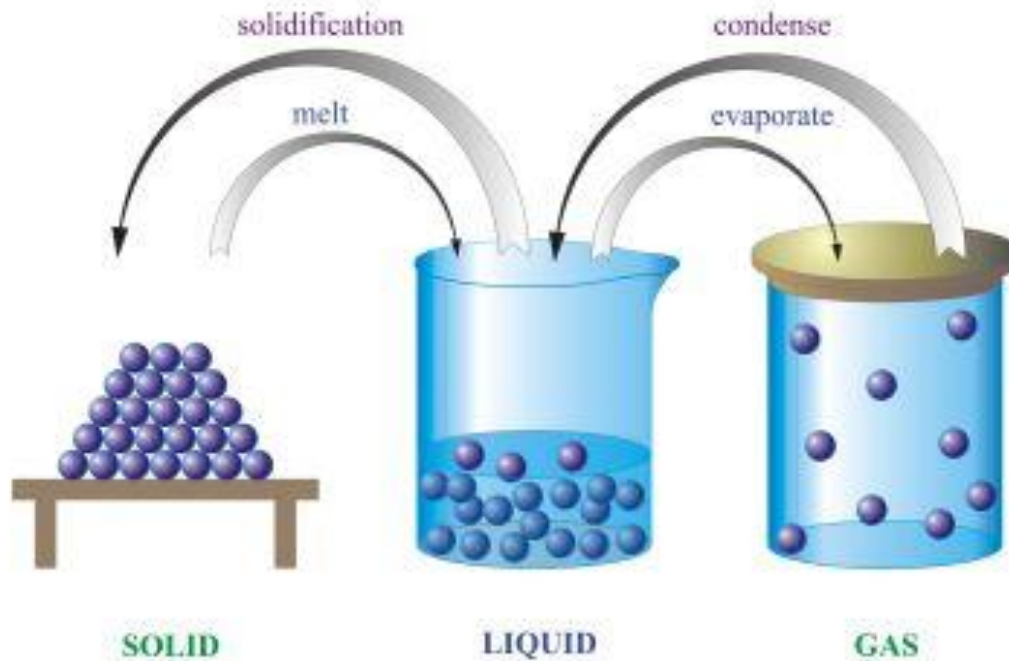
An entity on Earth consists of materials which could be in one of the four states such as:



Example of Transitions among States

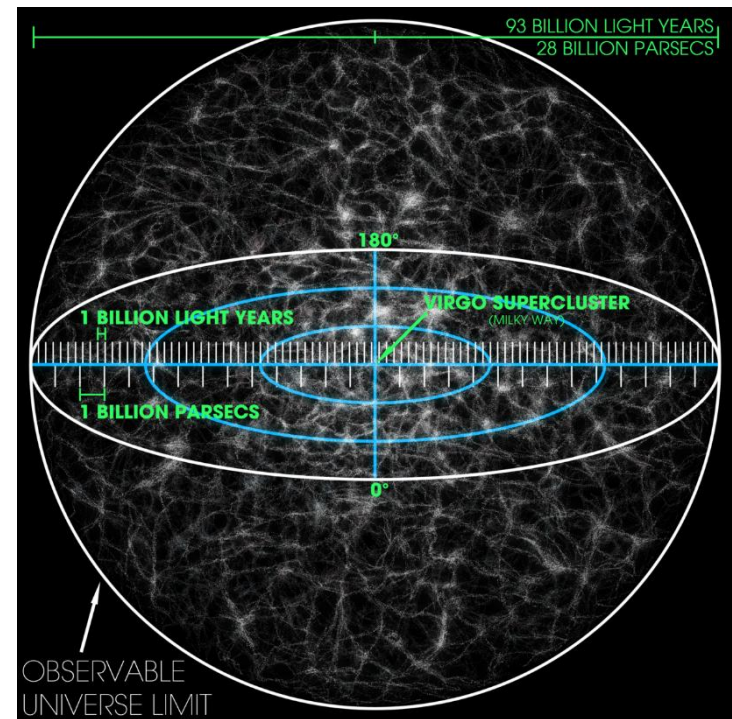
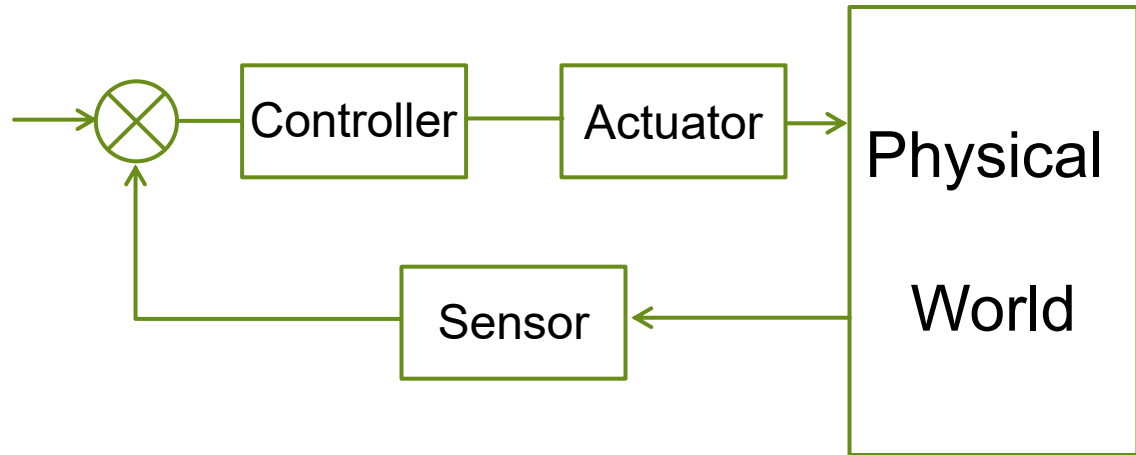


Example of Transitions among States



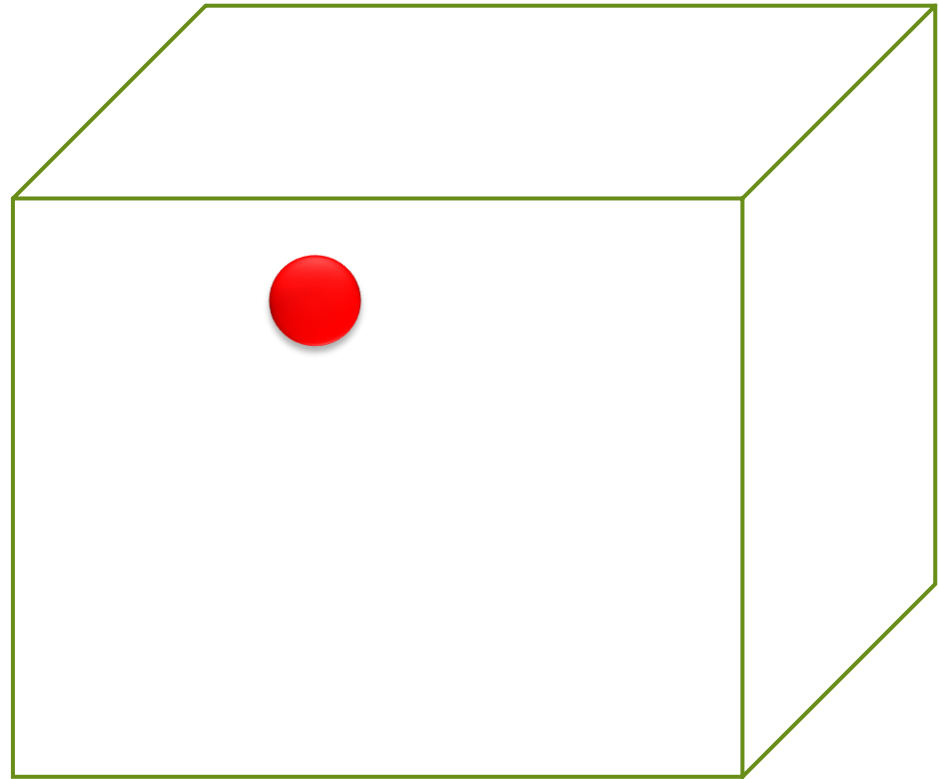
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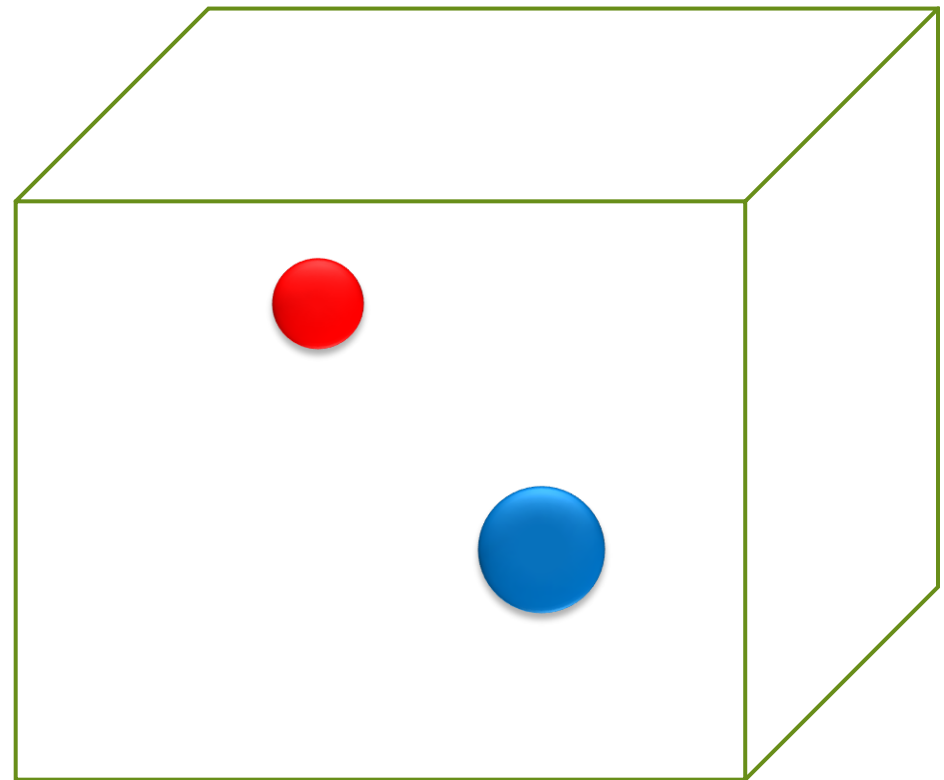
Physical World with 1 Floating Entity

- ▶ There are physical properties about the entity.
- ▶ There is no interaction.



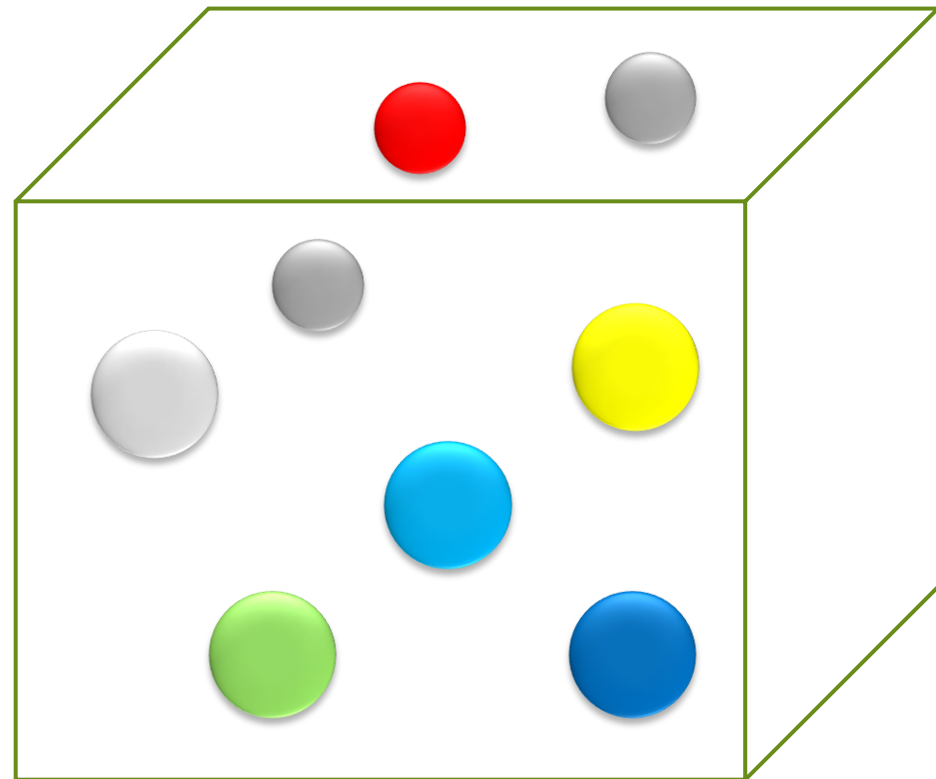
Physical World with 2 Entities

- ▶ There are physical properties about these two entities.
- ▶ There are mutual interactions.
- ▶ Interactions create relationships.
- ▶ Relationships include:
 - ▶ Membership
 - ▶ Family
 - ▶ Category, etc



Physical World with N Entities

- ▶ There are physical properties about these entities.
- ▶ There are multiple interactions.
- ▶ Interactions create relationships.
- ▶ Relationships include:
 - ▶ Membership
 - ▶ Family
 - ▶ Category, etc.



Definition of Properties

- ▶ The physical quantities, which are related to the self-existence of a physical entity, are called properties.



Example

A human being

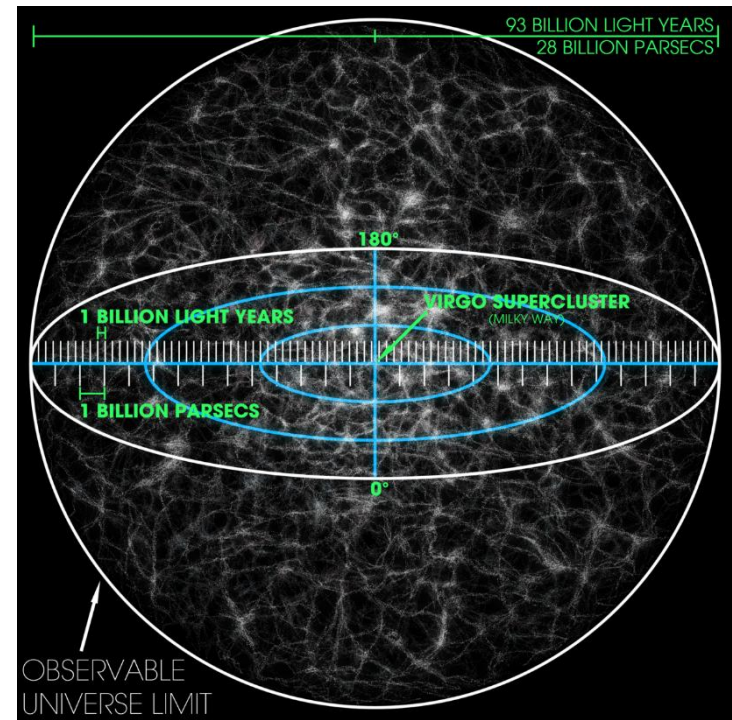
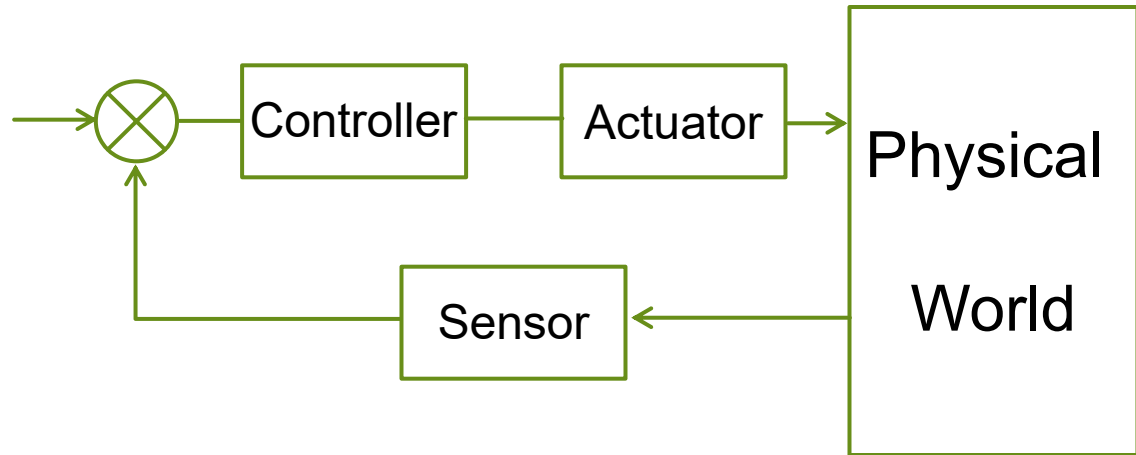
- ▶ Mass
- ▶ Height
- ▶ Gender
- ▶ Volume
- ▶ etc

A car

- ▶ Mass
- ▶ Length
- ▶ Width
- ▶ Colour
- ▶ etc

Outline

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Self-Existence of 1 Grounded Entity

- ▶ The existence of an entity depends on its properties.
 - ▶ Physical/chemical/biological/social/etc properties
- ▶ The existence of an entity is subject to space constraint.
 - ▶ Position
 - ▶ Dimension
 - ▶ Direction
- ▶ The existence of an entity is subject to time constraint.
 - ▶ Life span
 - ▶ Self-behaviors as a function of time



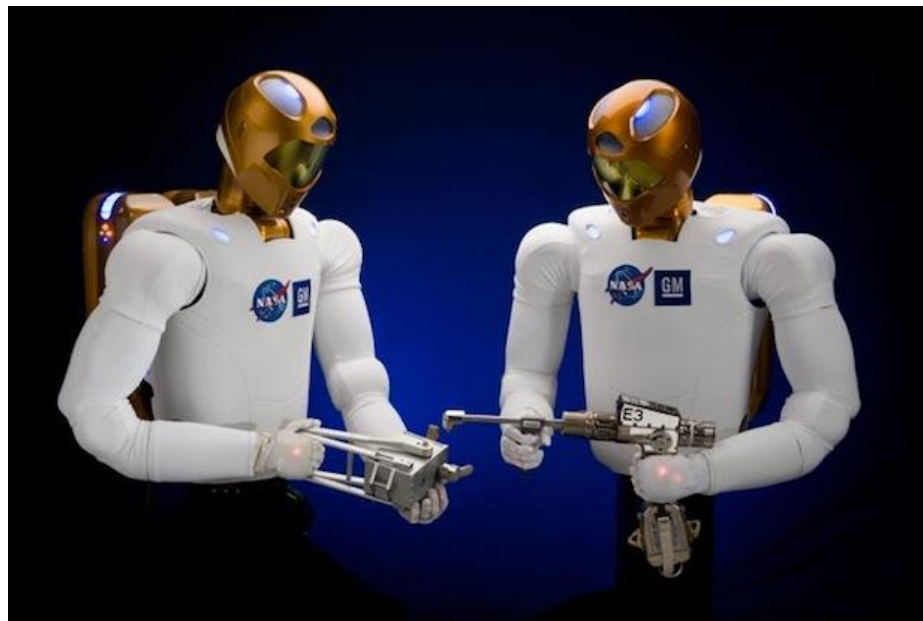
Co-Existence of N Entities

- ▶ The co-existence involves interactions among N entities.
 - ▶ Action
 - ▶ Reaction
- ▶ The co-existence is subject to space constraint.
 - ▶ Configuration
 - ▶ Topology
- ▶ The co-existence is subject to time constraint.
 - ▶ Self- and interactive behavior as a function of time
 - ▶ Event as a function of time
 - ▶ Episode as a function of time



Definition of Constraints

- ▶ The physical quantities, which are related to the co-existence of multiple physical entities, are called constraints.



Example

A human being

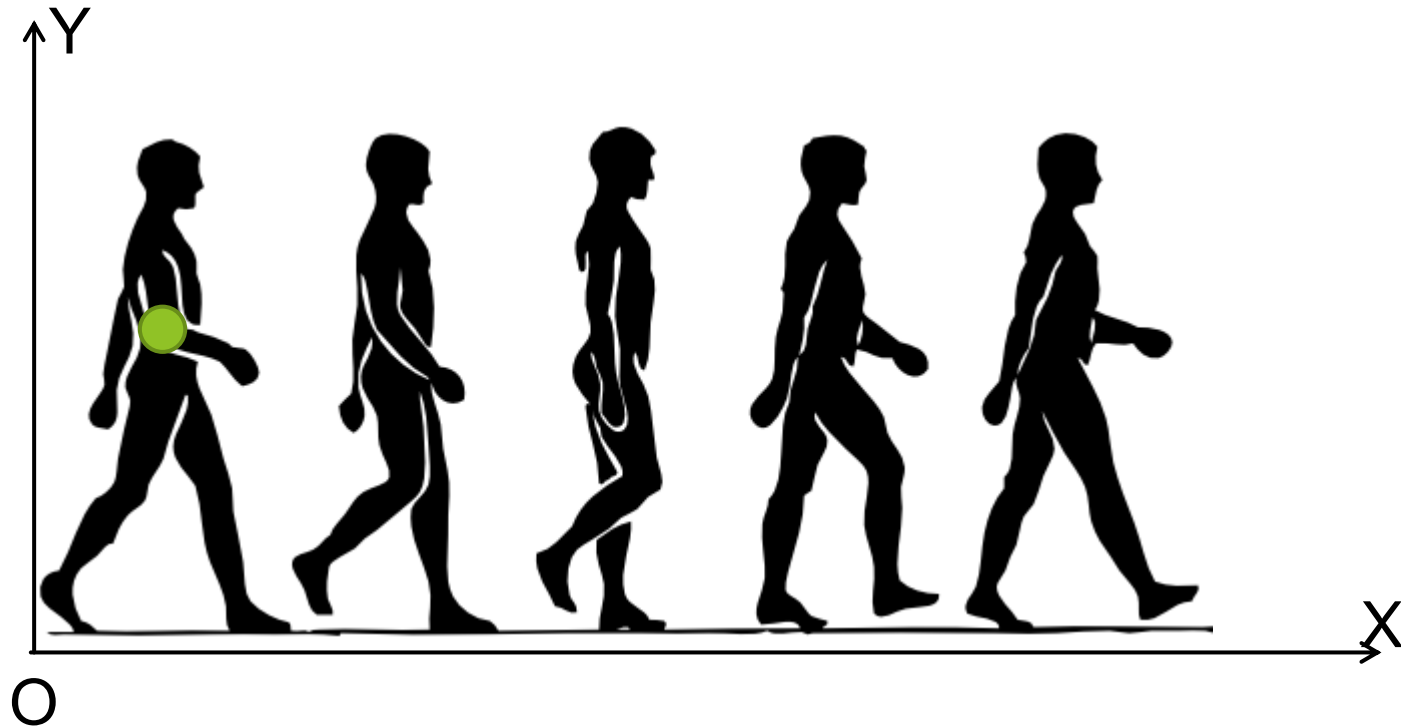
- ▶ Weight
- ▶ Position
- ▶ Age
- ▶ Degrees
- ▶ etc

A car

- ▶ Weight
- ▶ Velocity
- ▶ Owner
- ▶ Price
- ▶ etc

Example of Spatial Constraints

- ▶ Position/Velocity/Acceleration/Weight (Gravity)



Example of Spatial Constraints

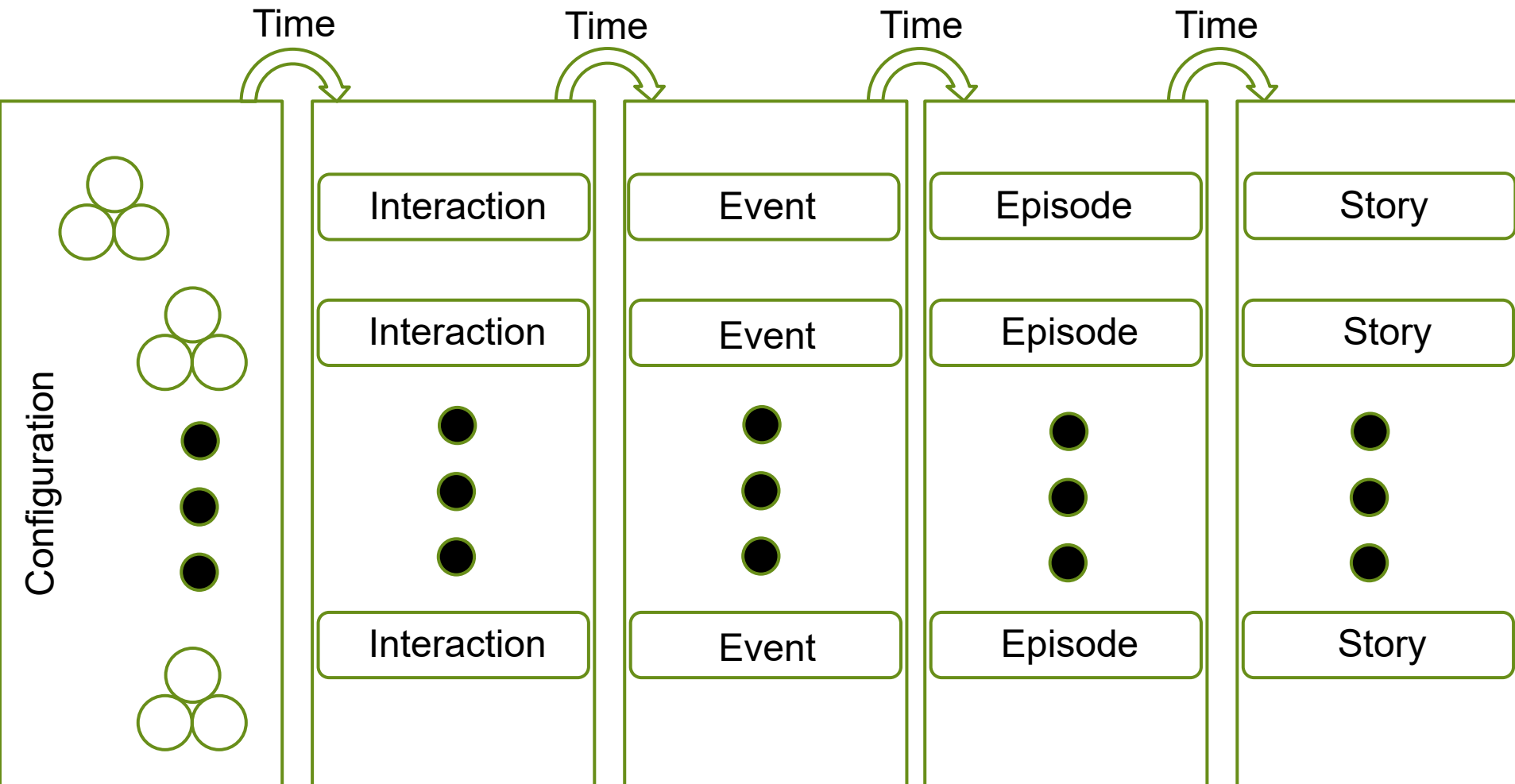


Definition of Behaviours

- ▶ Evolutions of properties and constraints as functions of time are called behaviours.



Dynamics Among N Entities



What is history? = a set of {stories}

Example of Dynamics among N Entities

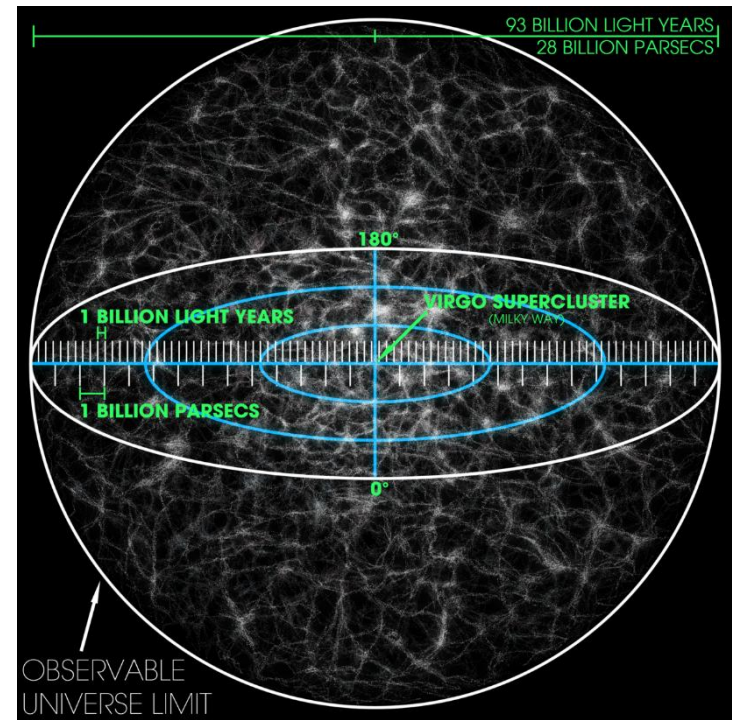
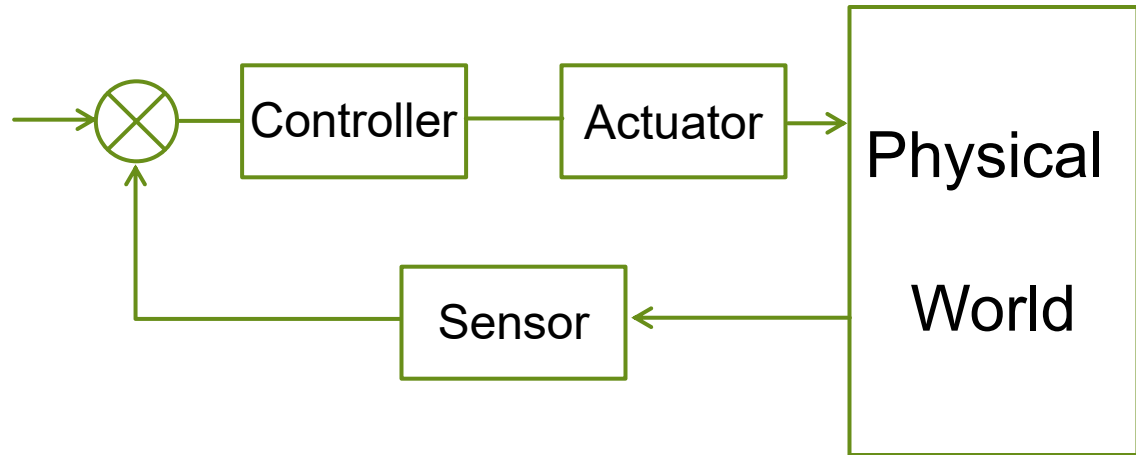


Are properties and constraints measurable?

Are they the manifestations of energies?

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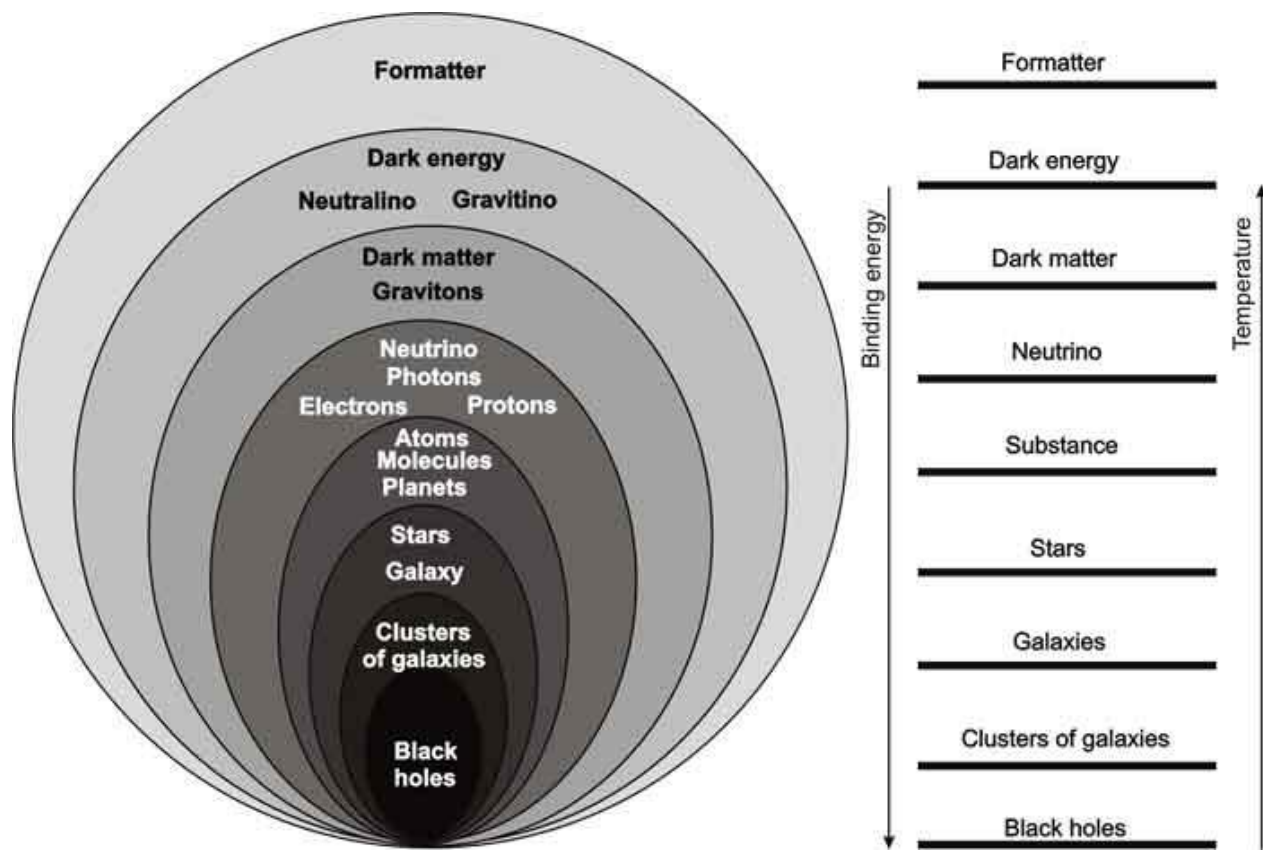
We are living in the physical world on Earth



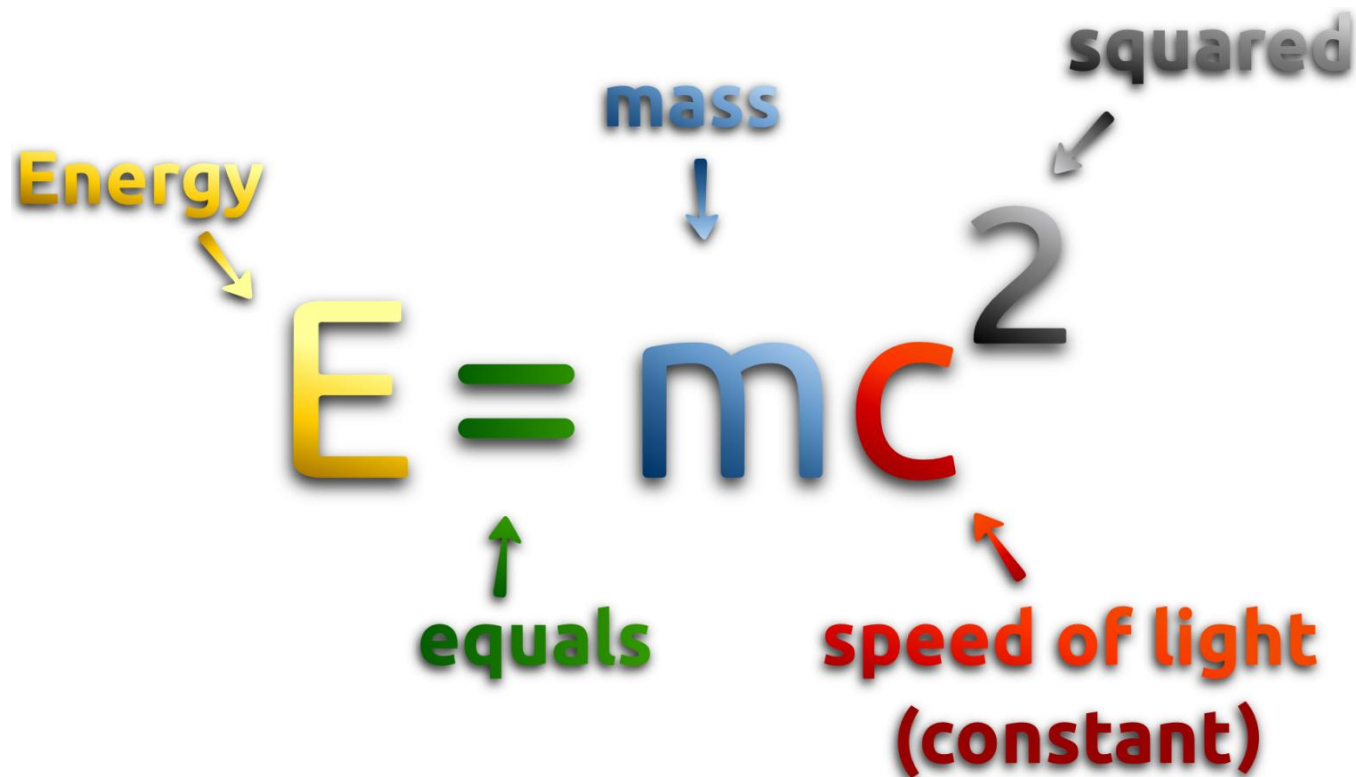
All existences on Earth are also manifestations of energies ...

► The popular energies include:

- Chemical energy
- Thermal energy
- Mechanical energy
- Electrical energy



For example, energy is manifested in the form of mass and speed



The diagram illustrates the equation $E = mc^2$ with the following labels and arrows:

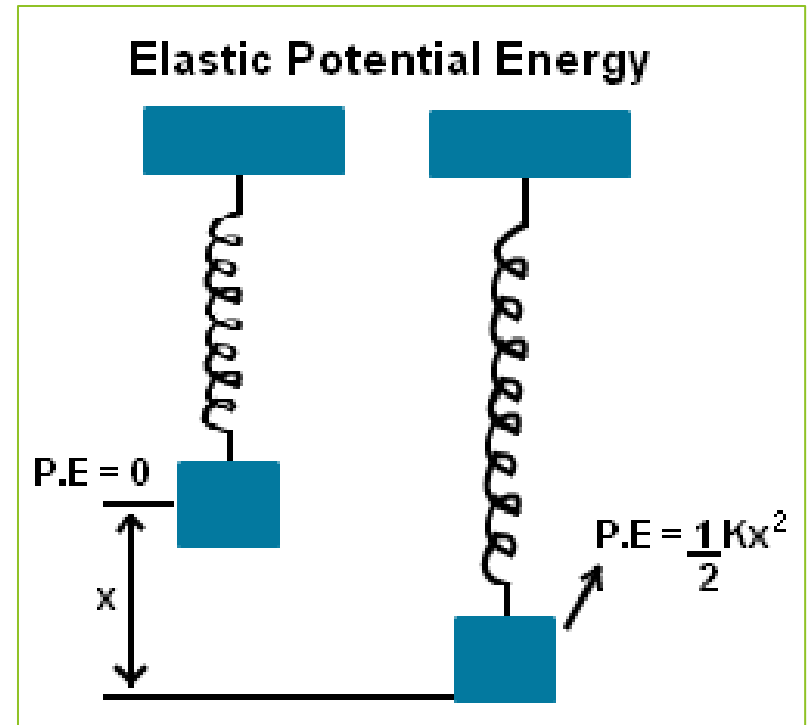
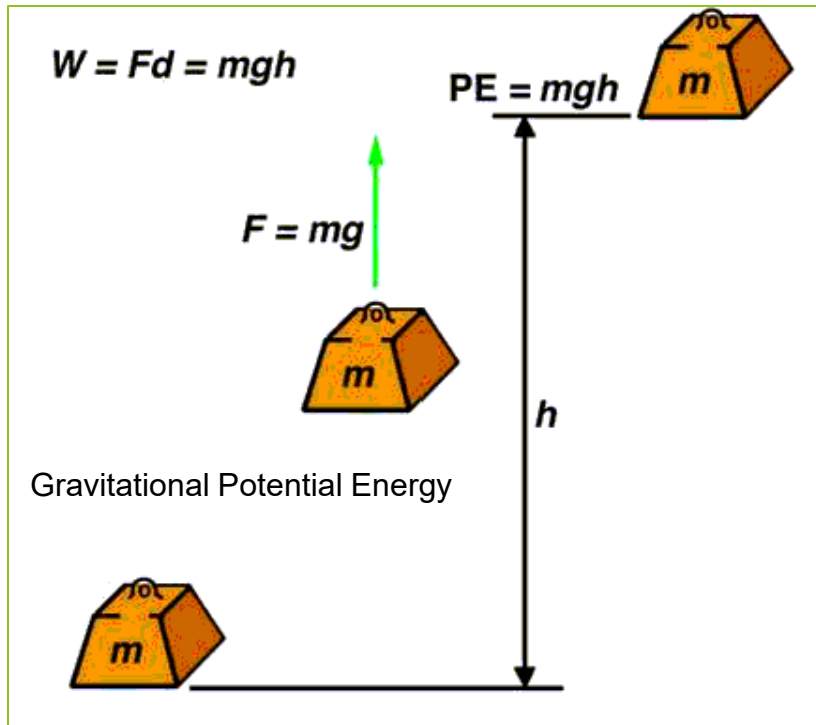
- Energy** (yellow text) with a yellow arrow pointing to the letter **E**.
- mass** (blue text) with a blue arrow pointing to the letter **m**.
- squared** (grey text) with a grey arrow pointing to the superscript **2**.
- equals** (green text) with a green arrow pointing to the equals sign **=**.
- speed of light (constant)** (red text) with a red arrow pointing to the letter **c**.

For example, energy is manifested in the form of mass and speed

$$E = \frac{1}{2} m \cdot v^2$$

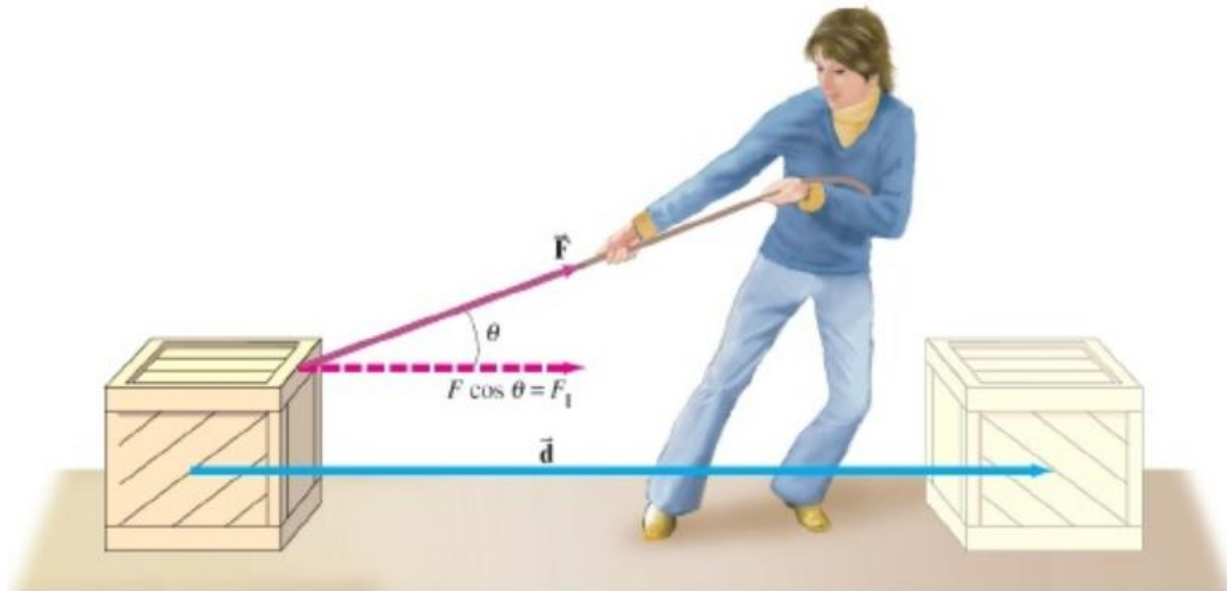


For example, energy is manifested in the form of mass and the relative height



For example, energy is also manifested in the form of force which makes a mass to undertake a displacement

$$W = Fd \cos \theta.$$

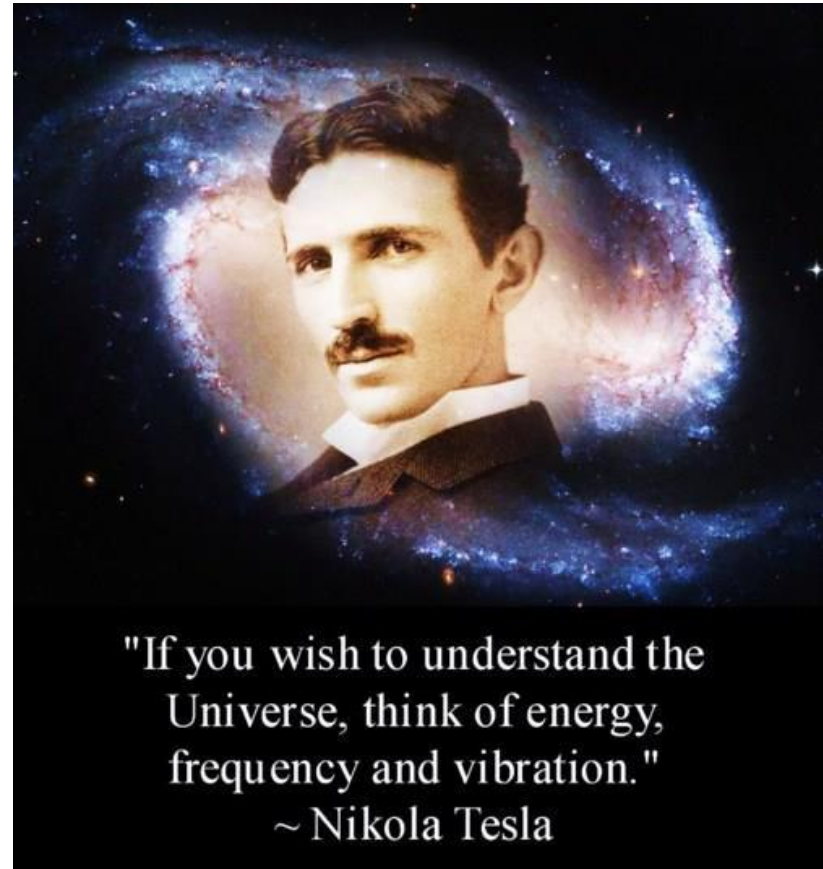


Definition of Physical Quantities

- ▶ Physical quantities refer to the **quantifiable properties** which are possessed by physical entities or physical systems.
- ▶ Physical quantities are manifestations of energies in the form of **signals**.

What is signal?

- ▶ Signal refers to the sum of periodic changes of values (this is the conclusion from Fourier Transform).
- ▶ Periodic changes of values are due to vibrations.
- ▶ Vibrations are due to transfers of energies.
- ▶ Periodic changes of values have different frequencies.



Vibration = Oscillation = Sine Wave

Example of Physical Quantities

Visible

- ▶ Length
- ▶ Height
- ▶ Volume
- ▶ Dimension
- ▶ Shape
- ▶ Colour

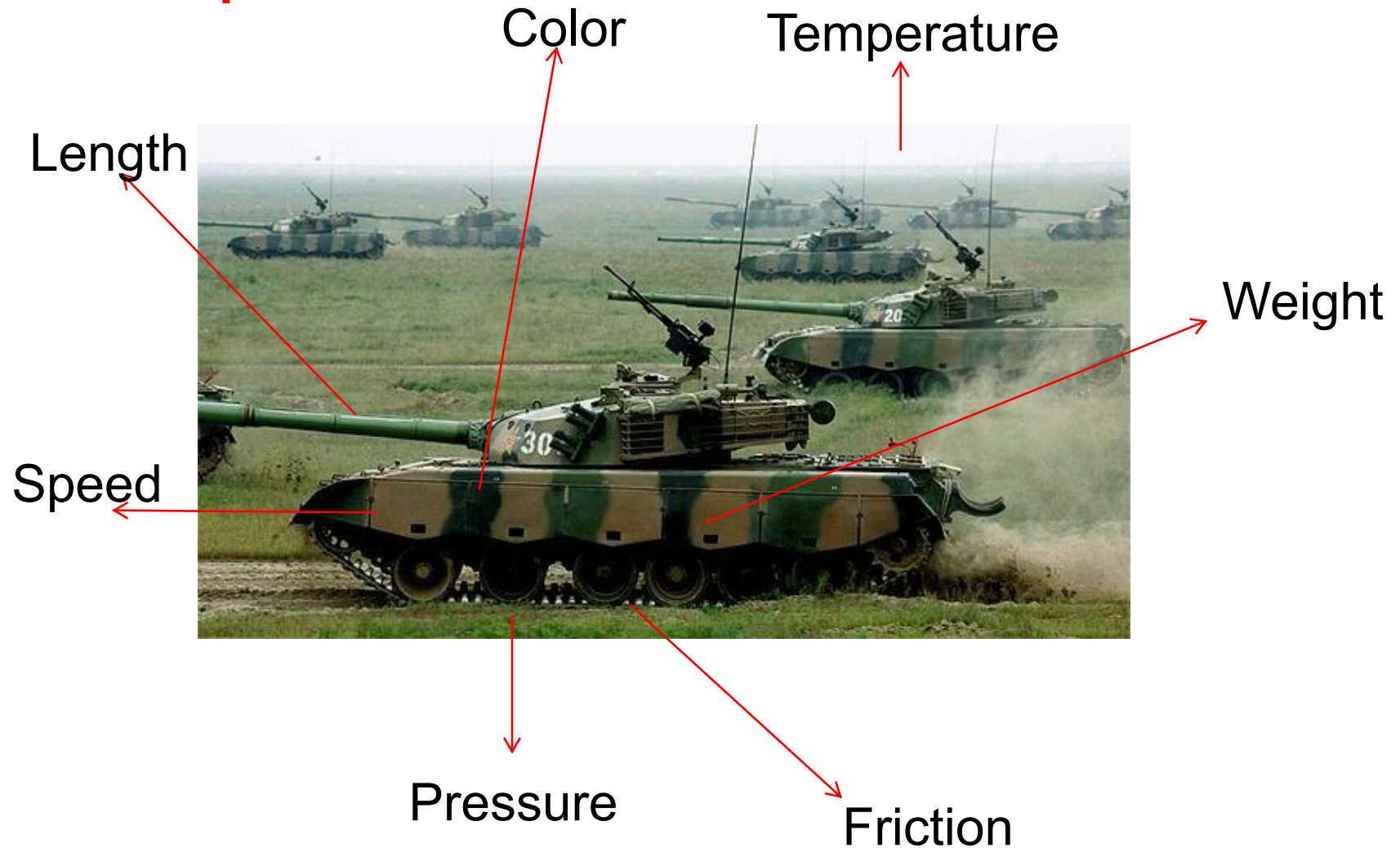
Signals

Invisible

- ▶ Energy
- ▶ Mass
- ▶ Weight
- ▶ Force
- ▶ Friction

Signals

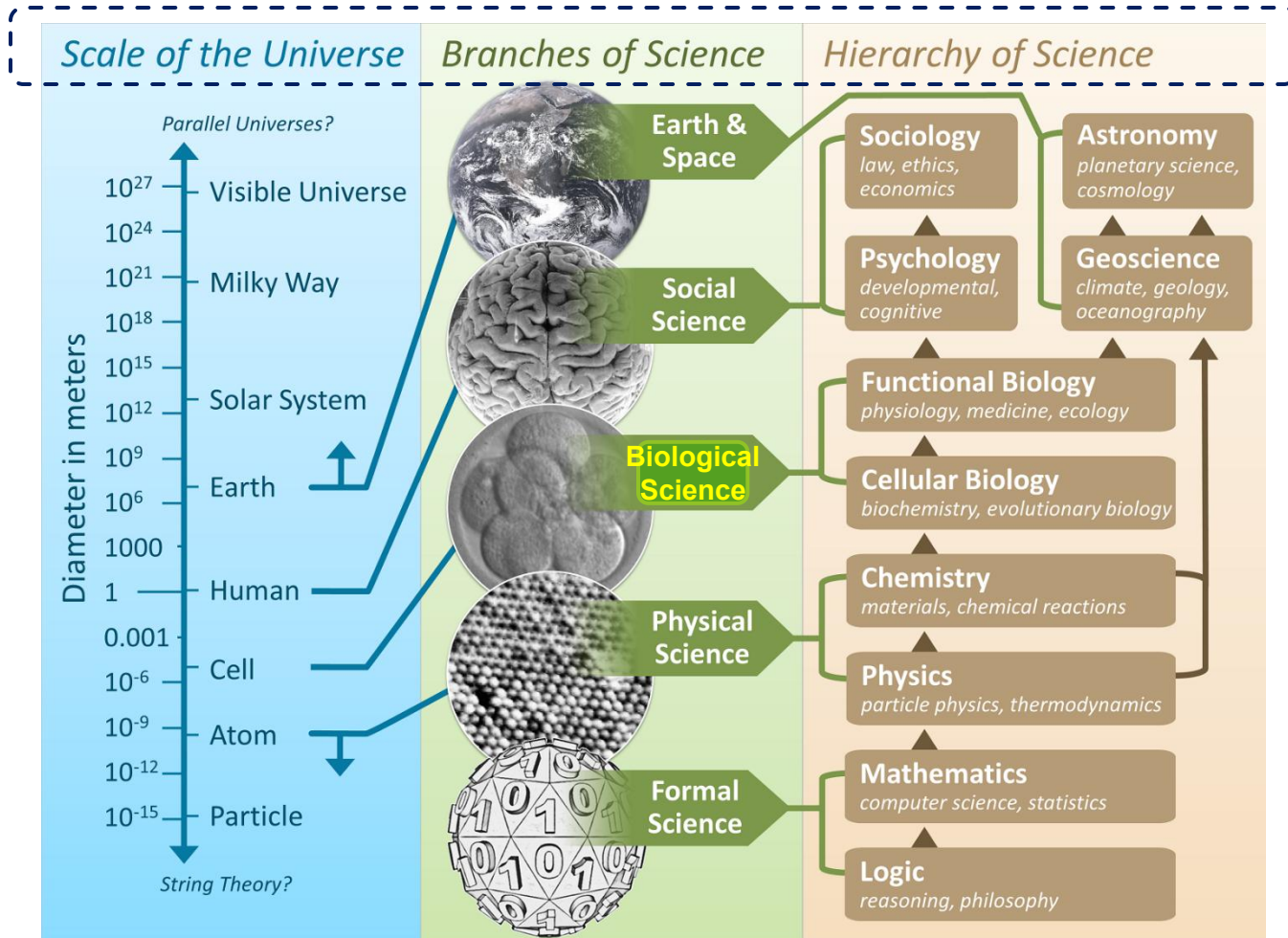
Example



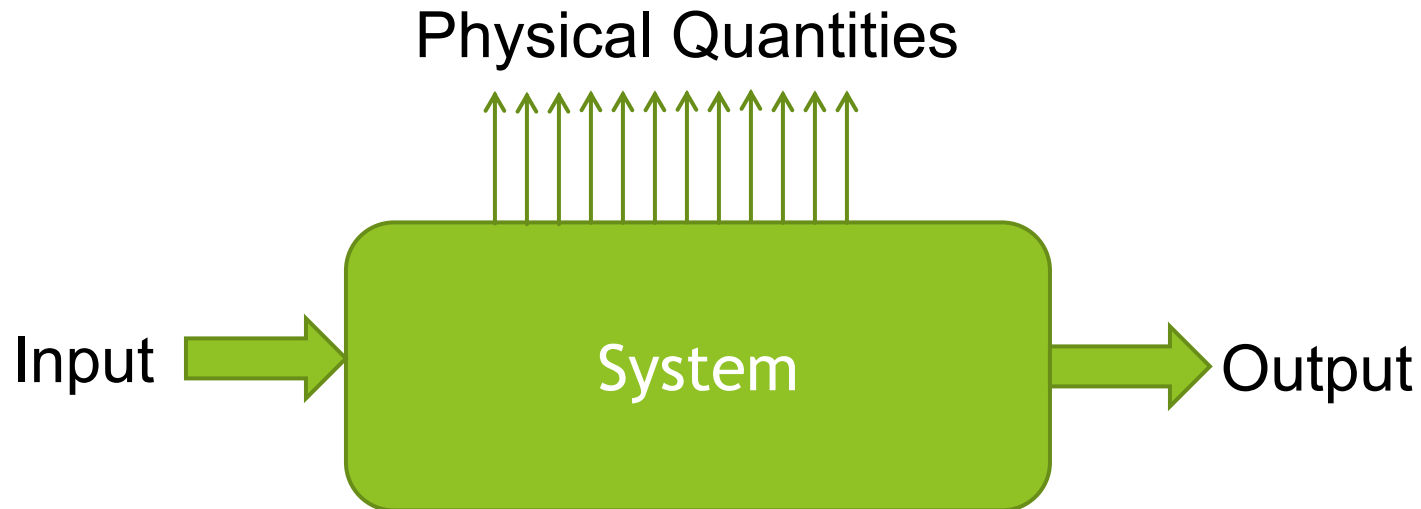
Two Missions of Human Beings on Earth Are ...

- ▶ To understand the world
 - ▶ Science (Discovery of Knowledge)
- ▶ To change the world
 - ▶ Engineering (Invention of Systems)

The understanding of the world is being undertaken in parallel in many disciplines ...

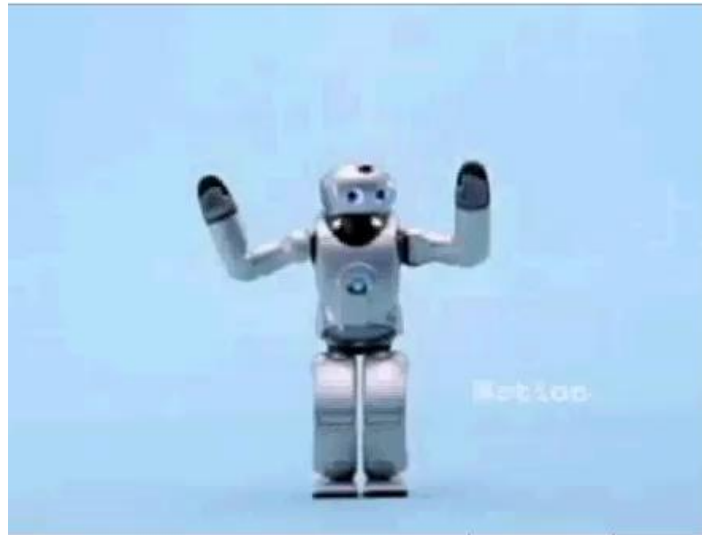


The effective way to understand the world is to adopt the viewpoint of systems



Example of a robot system

A system has input



A system has output



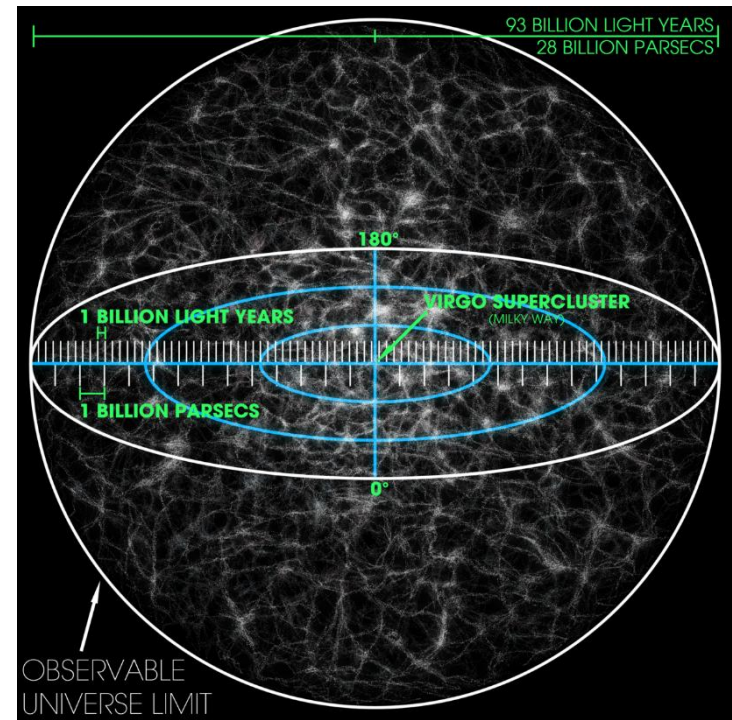
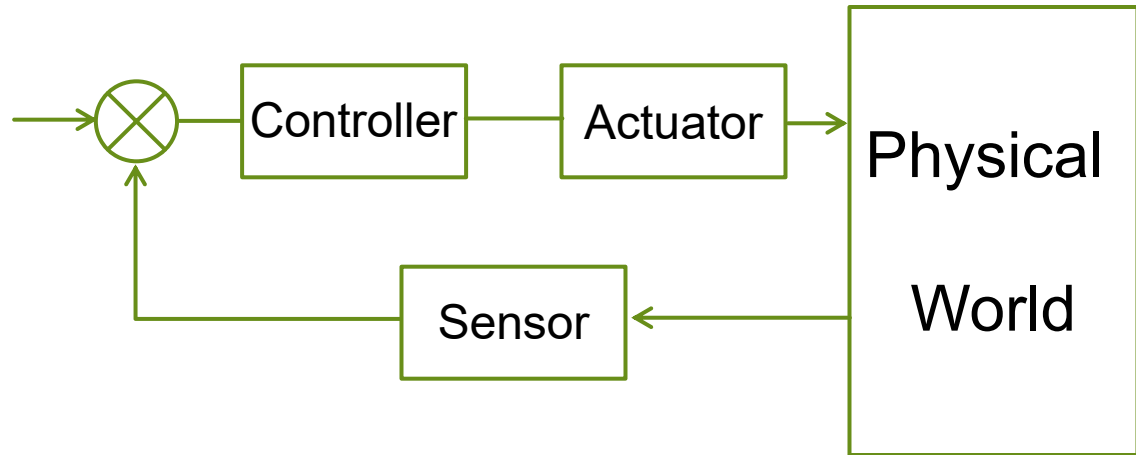
How to Do Measurement?

In general, there are two ways of undertaking direct measurements:

- ▶ Measurement by **Comparison**
 - ▶ Which makes use of references
- ▶ Measurement by **Sensing**
 - ▶ Which makes use of transducers

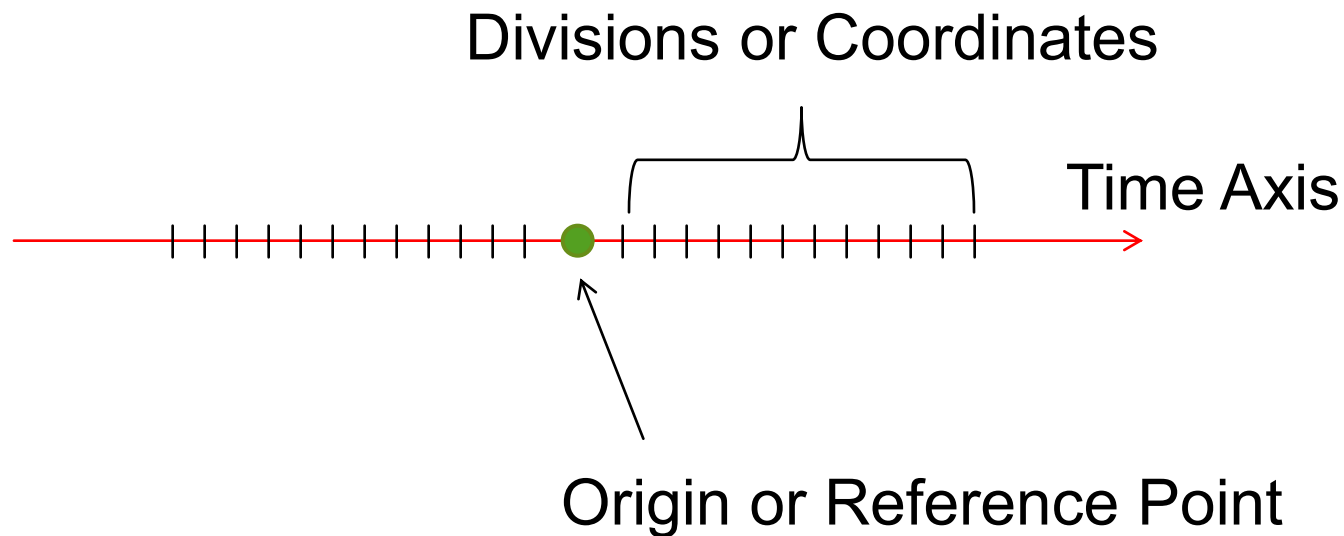
Outline

- ▶ Physical Systems
- ▶ Physical Entities
- ▶ Properties of Physical Entities
- ▶ Constraint of Physical Entities
- ▶ Physical Quantities
- ▶ Measurement by Comparison
- ▶ Measurement by Sensing

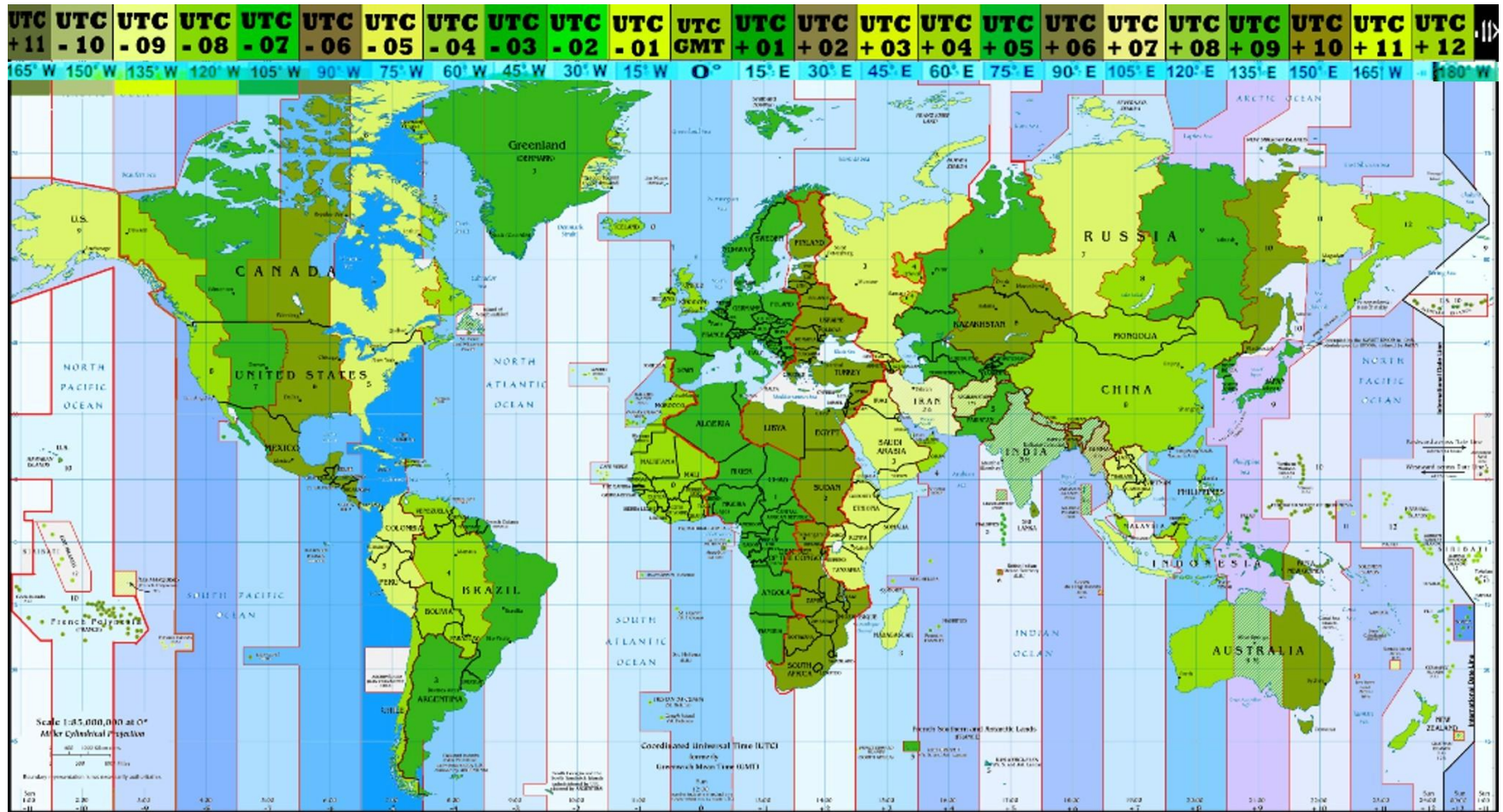


Reference of Time

- ▶ The value along an axis is called a coordinate.



Reference of Time Using 24 Meridians or Time Zones with 0 at Greenwich in England

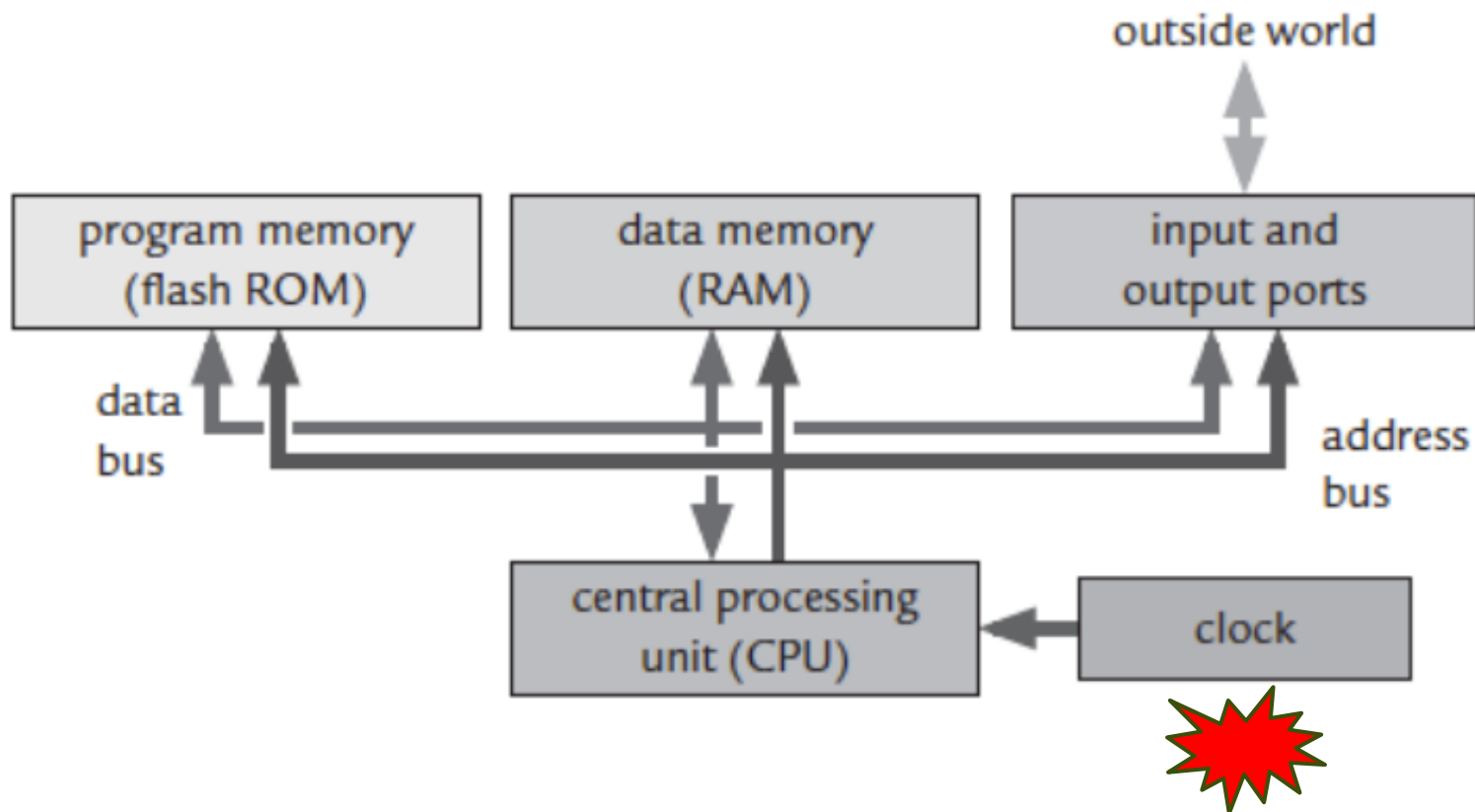


Example of Using Clock in Each Time Zone as Reference of Time

- ▶ Clock
- ▶ Always increasing



Example of Using Microprocessor's System Clock as Reference of Time



Unit of Time

- ▶ The present standard of time unit was adopted in 1967.
- ▶ The Cesium atom can transit between two sets of states at two lowest levels of energy.
- ▶ When bombarded by microwaves of certain frequency, Cesium atoms undergo transition from one state to another state with radiation.
- ▶ One second is equal to the duration in which 9,192,631,770 cycles of such radiation.

Units of Measurement of Time

▶ Time:

- ▶ Century
- ▶ Decade
- ▶ Year
- ▶ Month
- ▶ Week
- ▶ Day
- ▶ Hour
- ▶ Minute
- ▶ Second



Basic unit of measurement of time

Reference of Mass



Unit of Mass

- ▶ A particular cylinder of platinum-iridium alloy is kept at International Bureau of Weights and Measures at Sevres, Paris, France.
- ▶ The mass of this cylinder is one Kilogram.

Units of Measurement of Mass

▶ Mass:

▶ Ton

▶ Kilogram ← Basic unit of measurement of mass

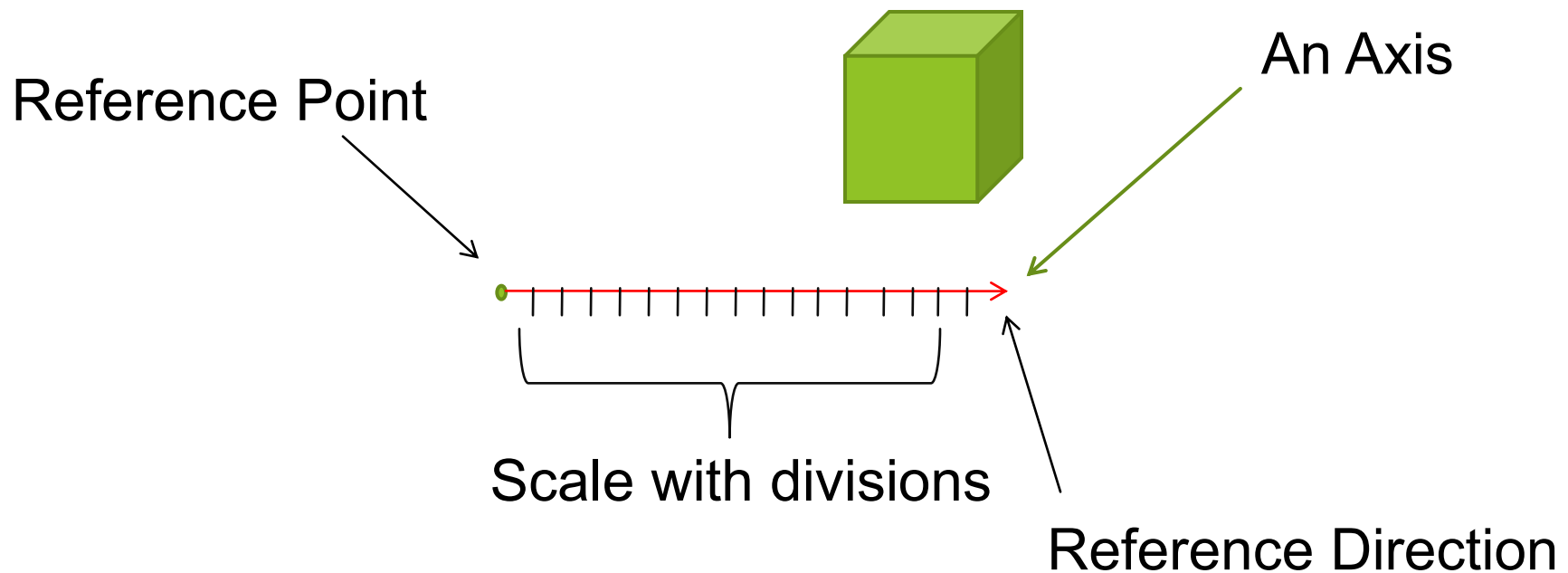
▶ Gram

▶ Milligram

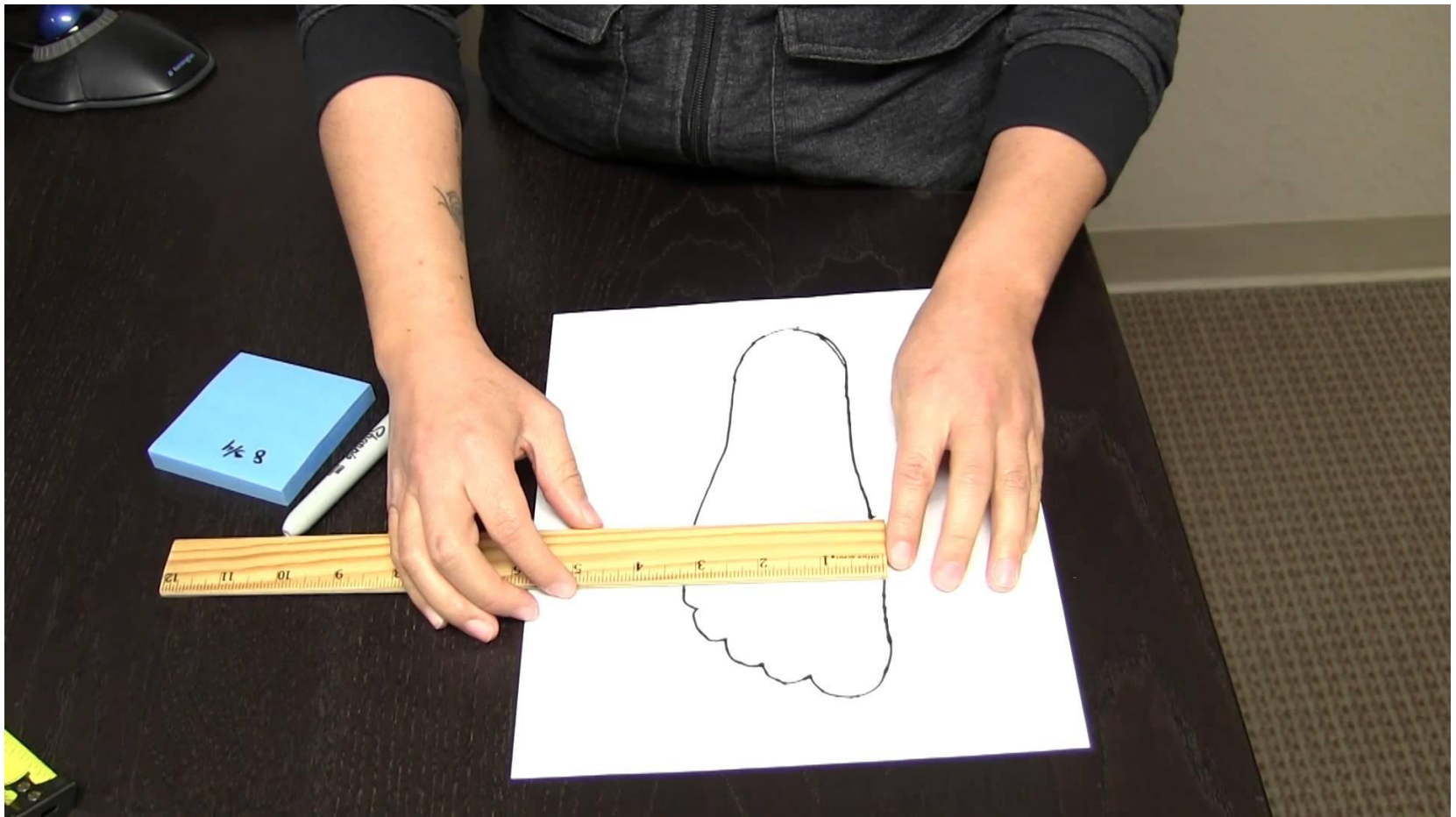
▶ Microgram

Reference of Space (i.e. Length)

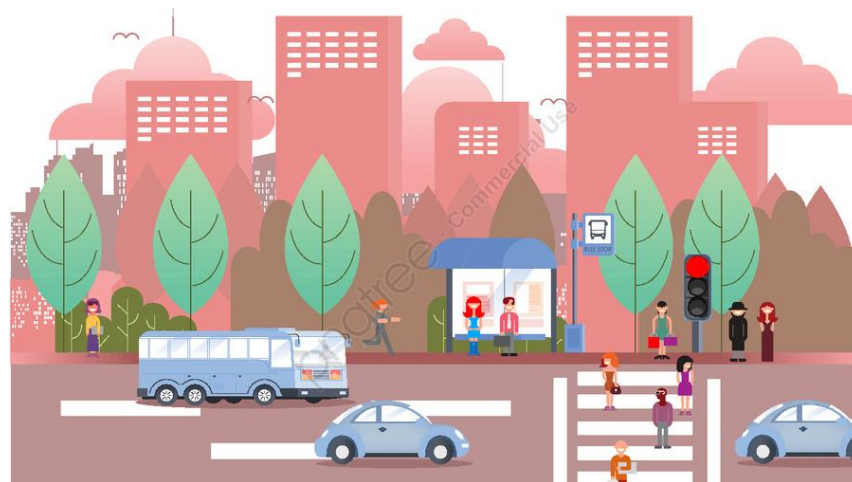
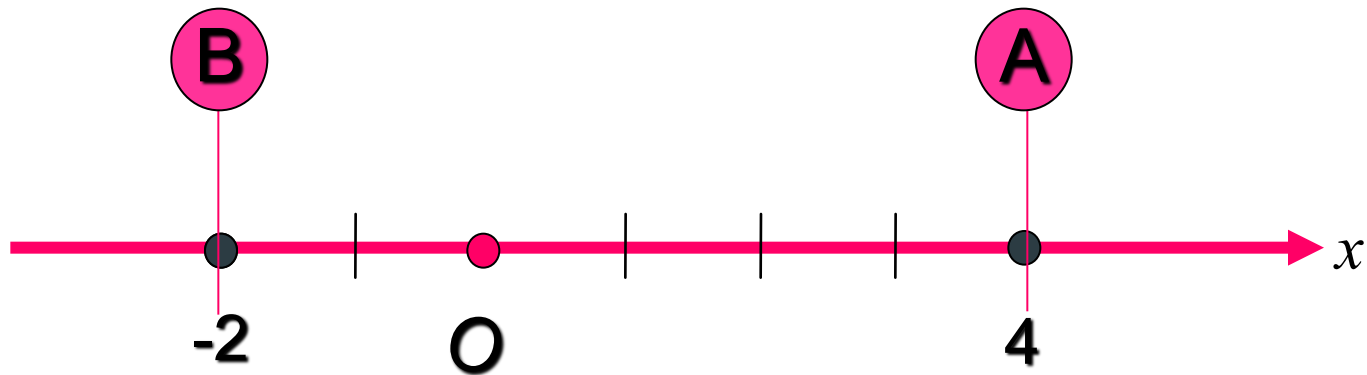
- ▶ The value along an axis is called a coordinate.



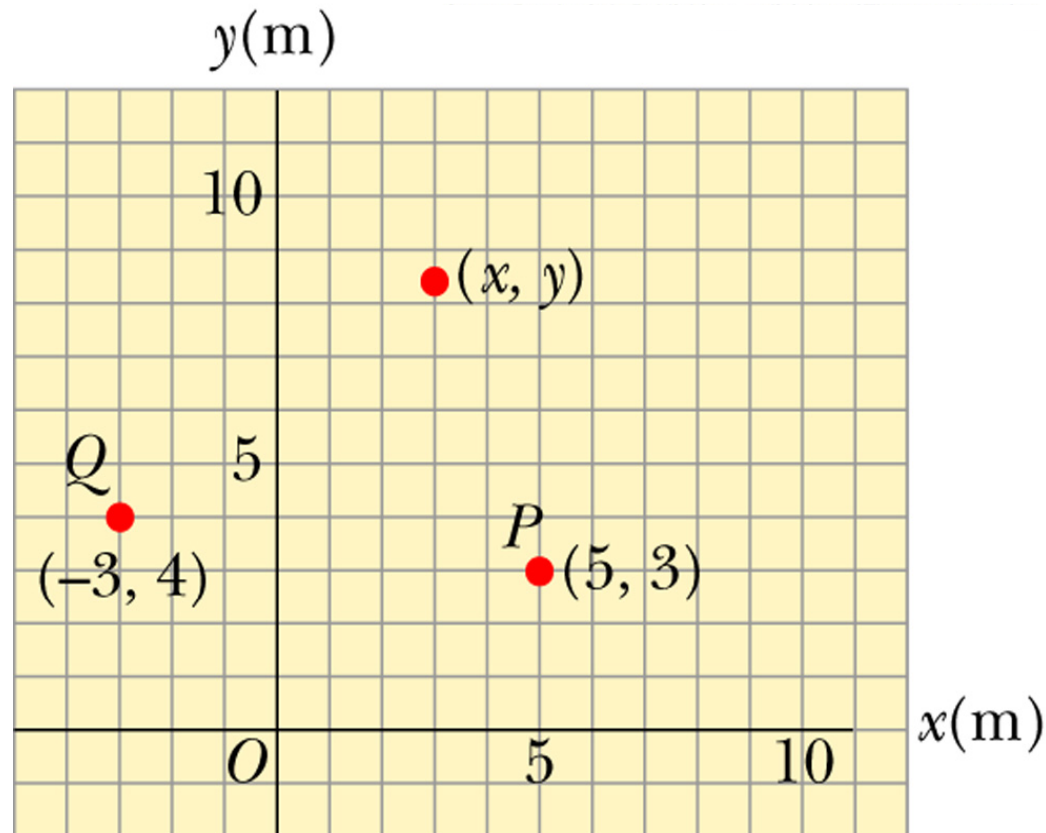
Example of Using Ruler as Reference of Length



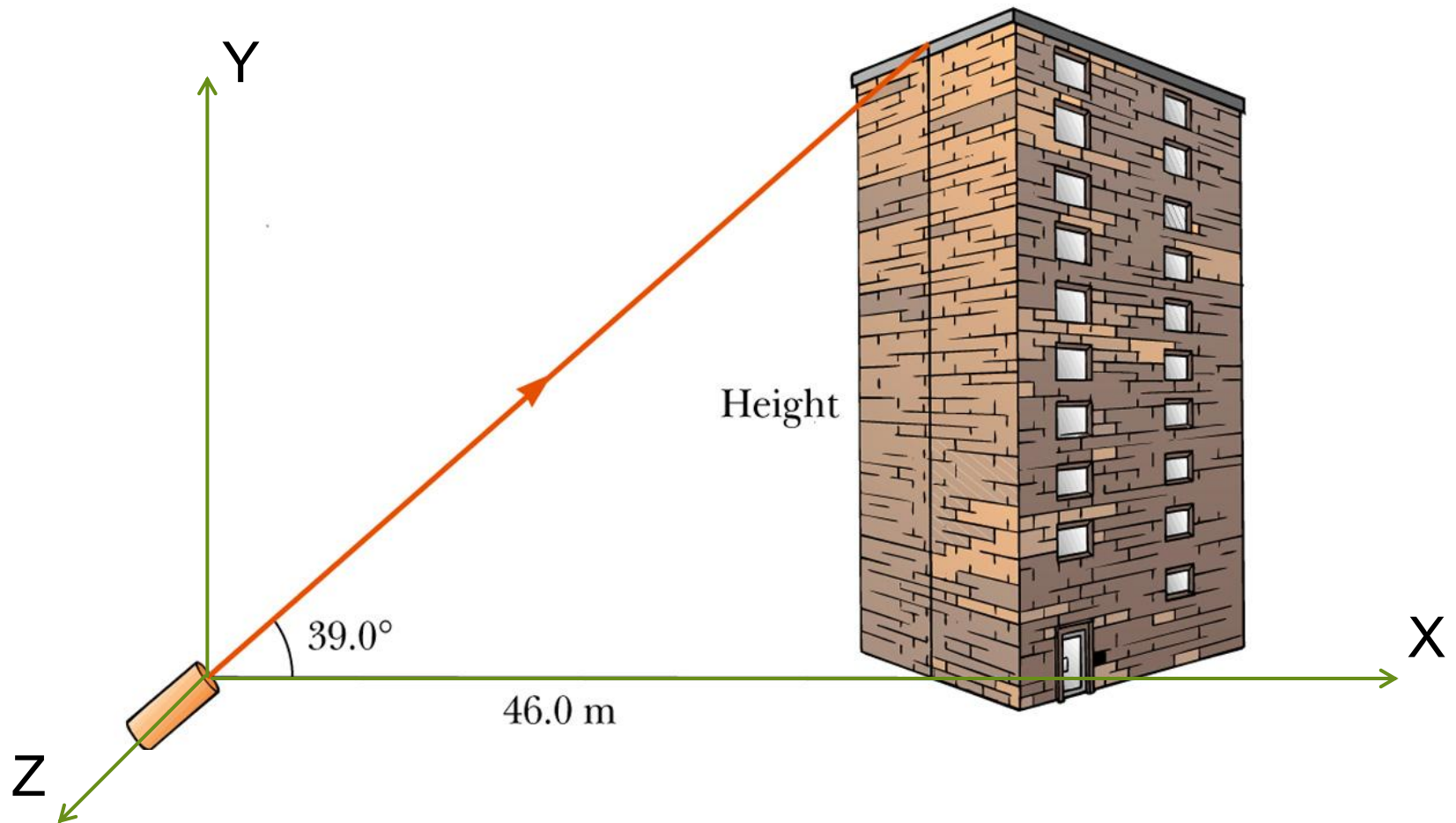
Example of Reference of One-Dimensional Space



Example of Reference of Two-Dimensional Space



Example of Reference of Three-Dimensional Space



Unit of Length

- ▶ The present standard was adopted in 1983.
- ▶ We define that the speed of light in a vacuum is 299,792,458 m/s.
- ▶ One meter is the distance travelled by light in a vacuum within $1/299,792,458$ seconds.

Units of Measurement of Length

- ▶ Length:
 - ▶ Kilometer
 - ▶ Meter ← Basic unit of measurement of length
 - ▶ Centimeter
 - ▶ Millimeter
 - ▶ Micrometer
 - ▶ Nanometer

Units of Measurement of Speed

- ▶ Speed:
 - ▶ Kilometer/hour
 - ▶ Meter/second ← Basic unit of measurement of speed

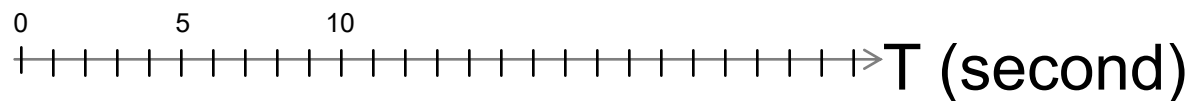


Example: What is the travelled distance by a ship within 2 minutes if it moves at a constant speed of 2 km/h?

$$T = 2 \times 60 = 120 \text{ seconds}$$

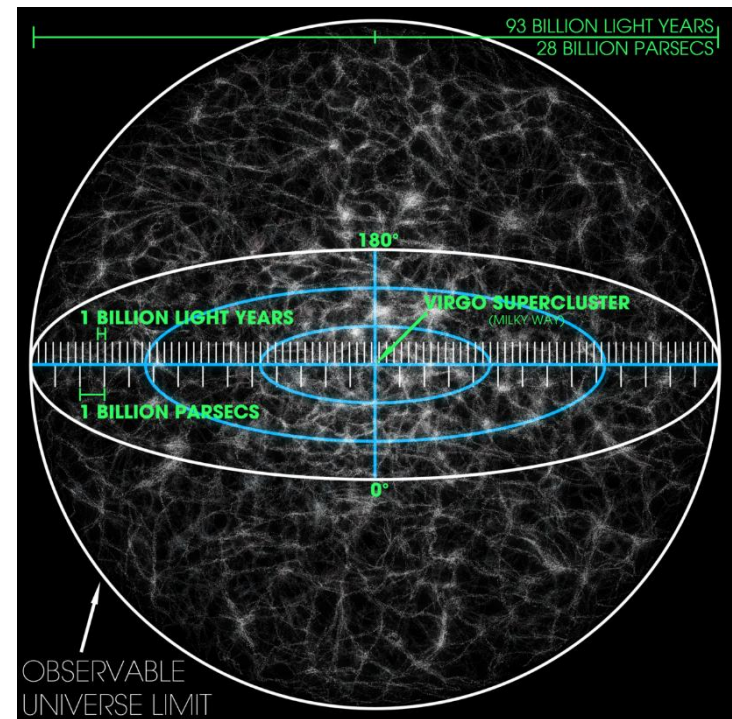
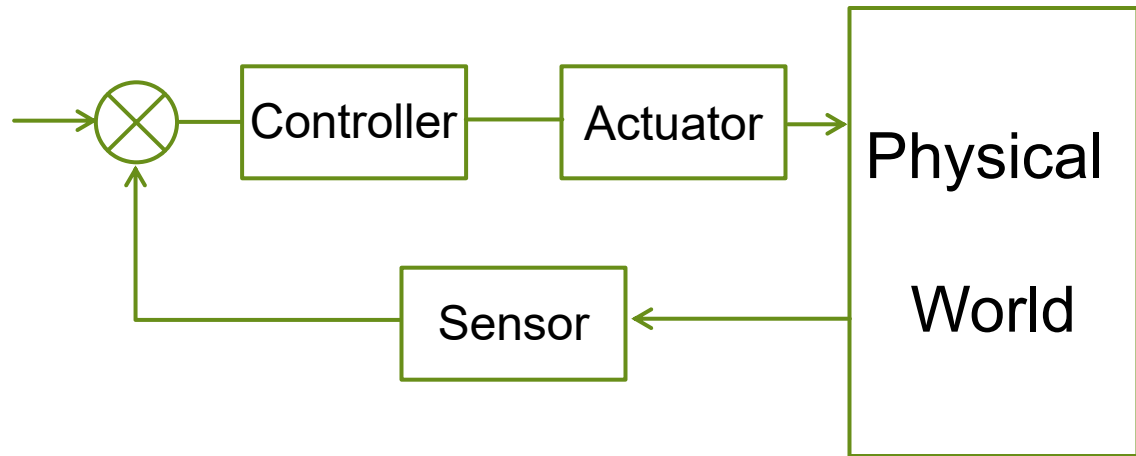
$$S = 2 \times 1000 / (60 \times 60) = 0.56 \text{ m/s}$$

$$D = S \times T = 0.56 \times 120 = 67.2 \text{ meters}$$



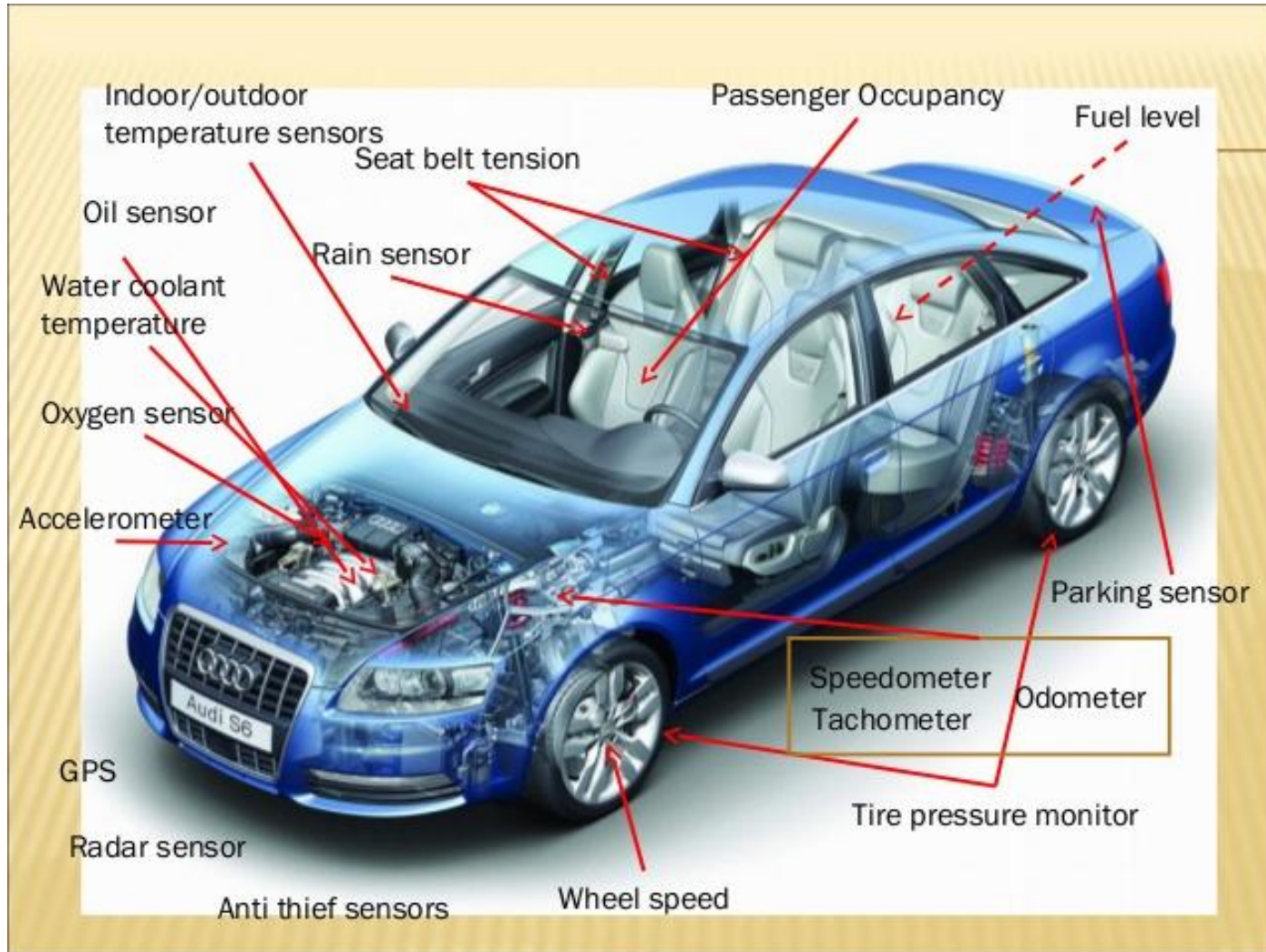
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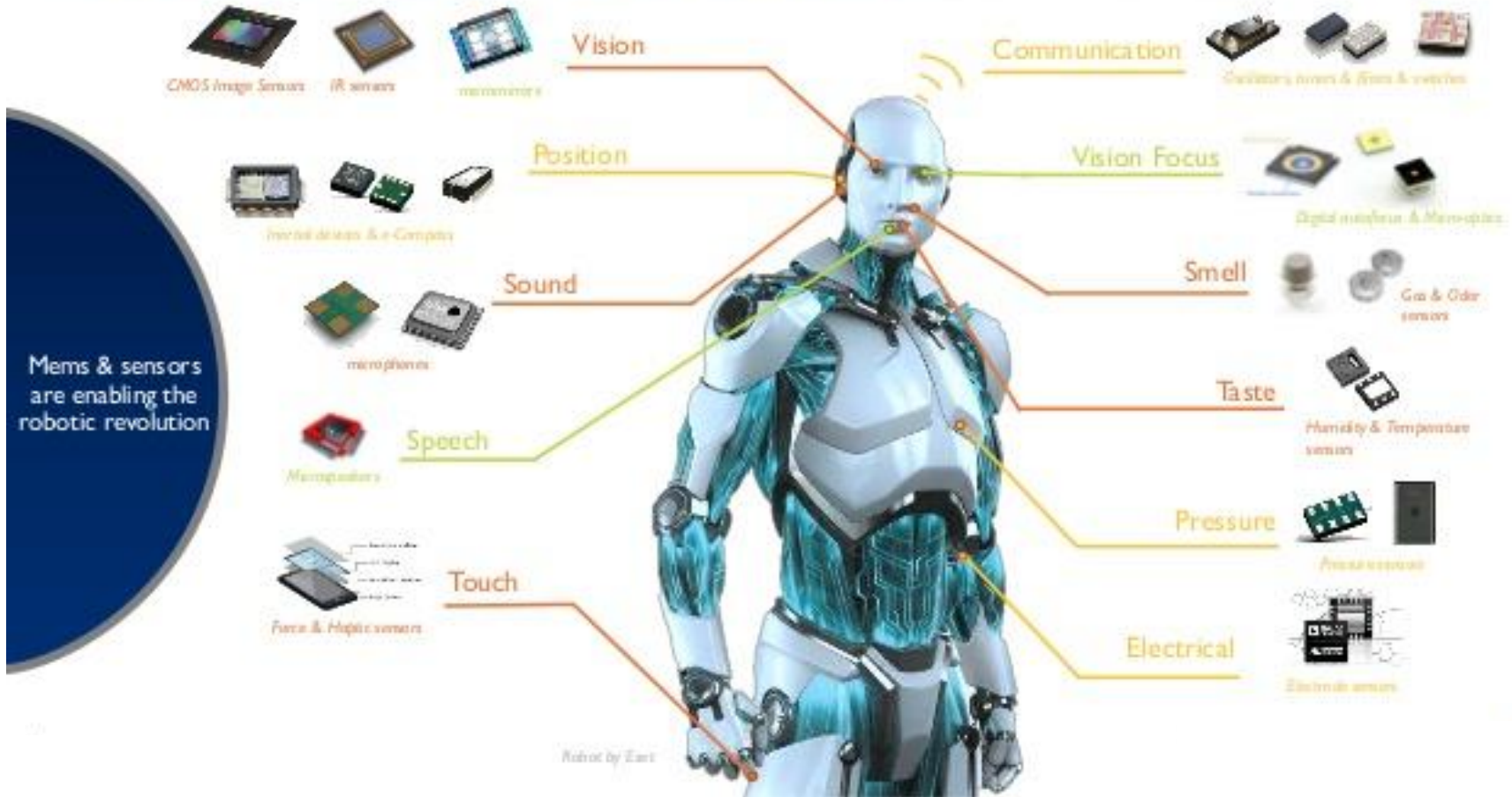
Measurement by Sensing

Why to do sensing and measurement?



Why to do sensing and measurement?

MEMS & SENSORS : BEYOND THE HUMAN SENSES...



Principle of Doing Sensing and Measurement

▶ Step 1:

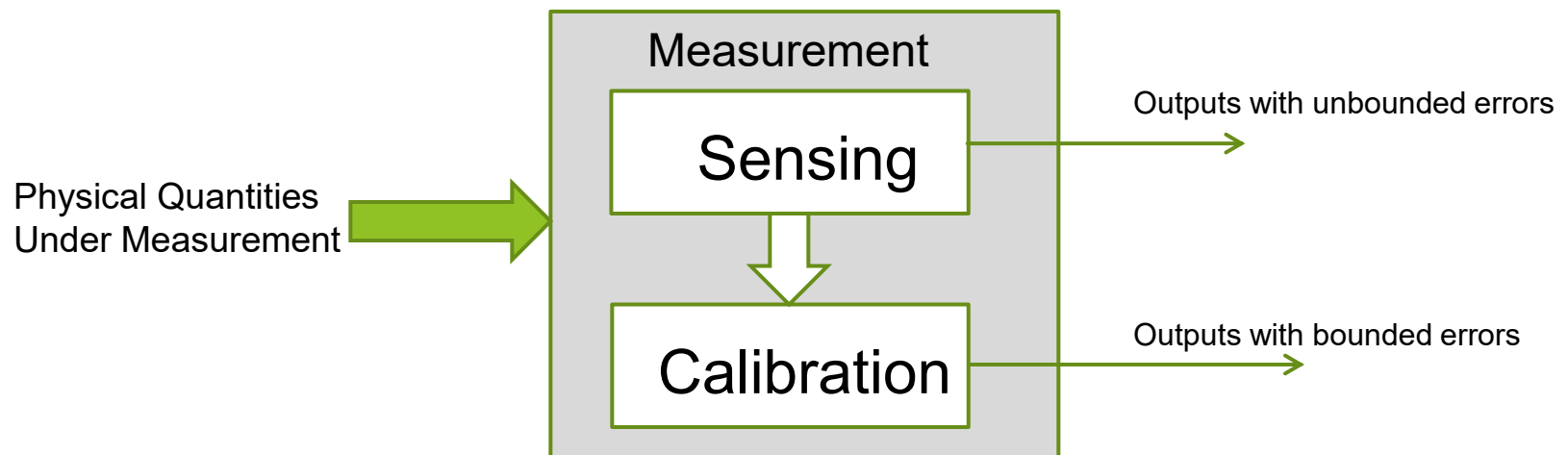
- ▶ To sense the values of physical quantities under measurement.

▶ Step 2:

- ▶ To calibrate the sensed values so as to obtain final outputs which must be exactly equal to the inputs (i.e. outputs with bounded errors).

What is the purpose of sensing?

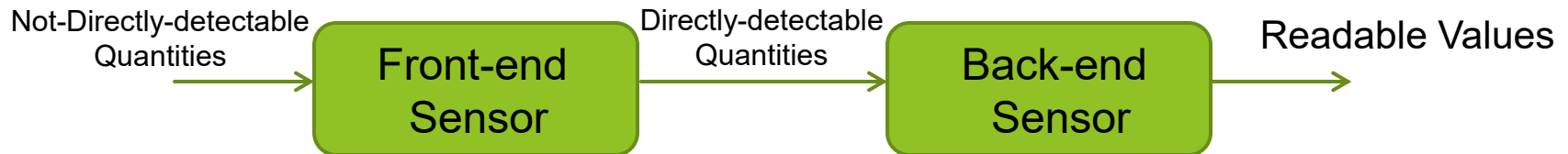
- ▶ The purpose of sensing is to detect the value of physical quantities and to output the corresponding **readable** values.



Challenges faced by sensing include:

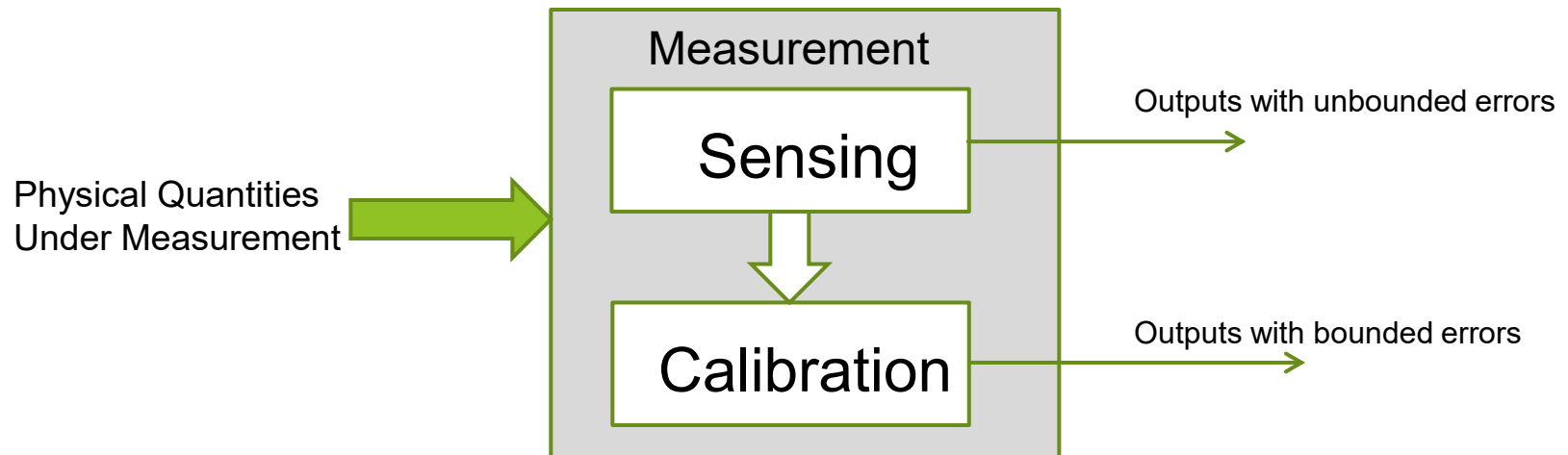
- ▶ How to sense the value of directly-detectable quantities?
- ▶ How to sense the value of not-directly-detectable quantities?

Solutions of Sensing



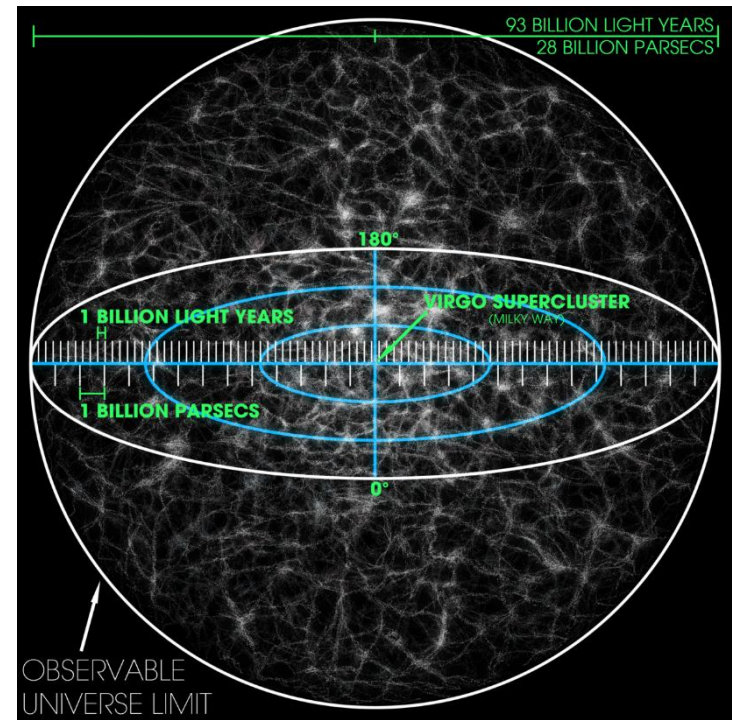
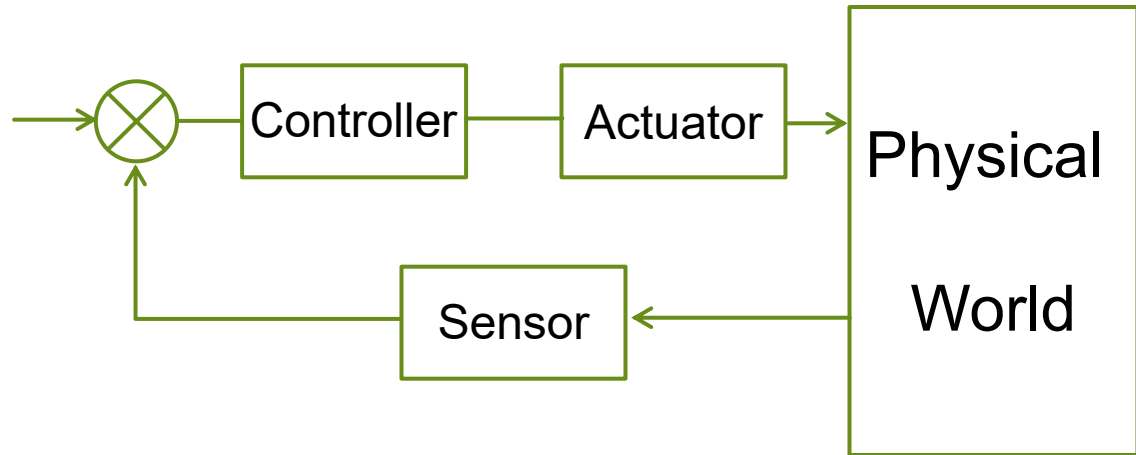
What is the purpose of calibration?

- ▶ The purpose of calibration is to transform the measurable values into the **readable and calibrated** values which must be **accurately** equal to the values under measurement.



Summary

- ▶ Physical Systems
- ▶ Physical Entities
- ▶ Properties of Physical Entities
- ▶ Constraint of Physical Entities
- ▶ Physical Quantities
- ▶ Measurement by Comparison
- ▶ Measurement by Sensing



Outline of Module 1

- ▶ Lecture 1:
 - ▶ Basics of Physical World
- ▶ Lecture 2:
 - ▶ Randomness of Physical World
- ▶ Lecture 3:
 - ▶ Basics of Conceptual Worlds
- ▶ Lecture 4:
 - ▶ Fuzziness of Conceptual Worlds





NANYANG
TECHNOLOGICAL
UNIVERSITY

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 1 Lecture 2

MA4822

Randomness of Physical World

Xie Ming, PhD (France)

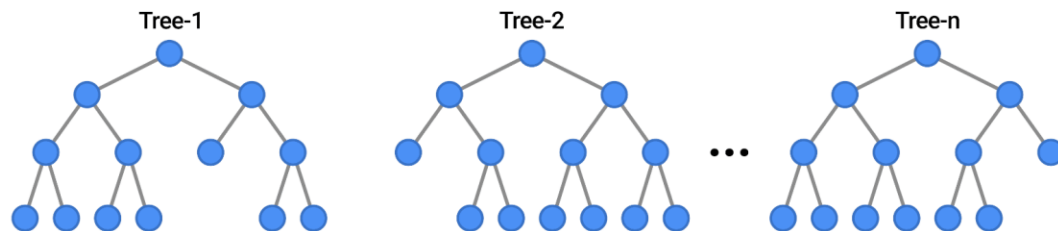
mmxie@ntu.edu.sg

<http://personal.ntu.edu.sg/mmxie>

Outline

- ▶ Concept of Randomness
- ▶ Statistics of Truth
- ▶ Operations of Truth
- ▶ Probability of Truth
- ▶ Determination of Probability
- ▶ Prediction Using Bayes' Theorem

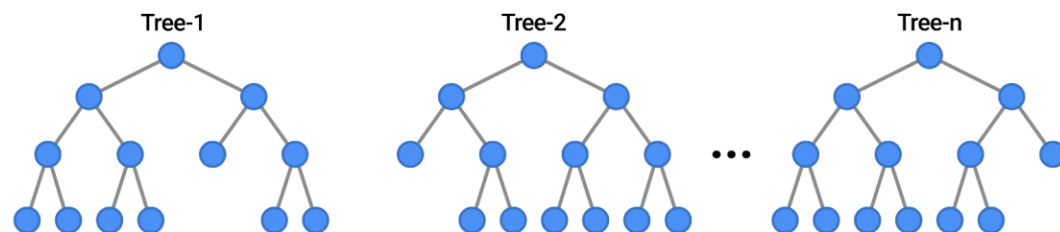
EXAMPLES



Outline

- ▶ Concept of Randomness
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EXAMPLES



We are living in a complex world

Abundance creates redundance which creates competition



We are also living in a competing world, which creates uncertainty ...

- ▶ Compete for success
- ▶ Compete for failure
- ▶ Compete for presence
- ▶ Compete for absence
- ▶ Compete for having ...
- ▶ Compete for being ...



Signals and noises are also competing for occurrence

Competitions cause randomness

For example,

- ▶ In a production line of automobile, the next finished car could be in red, white, green, yellow, dark or silver color.
- ▶ At a gate of a university campus, the next person entering the gate could be a male student, a female student, a male faculty, or a female faculty.
- ▶ Today could be a raining, cloudy or sunny day.
- ▶ **Signals/noises could compete for input to sensors.**

Example of competition by noises inside measurement and sensing systems

- ▶ There are multiple sources of noises with different magnitudes.
- ▶ These noises compete for occurrence.
- ▶ The outcome of measurement will be:

$$V_m = V_t + \Delta n$$

Measured Value

True Value

Variation Caused by Noise

Definition of Randomness

- ▶ Randomness refers to the chance of occurrence of truths (e.g. at input of sensing systems).
- ▶ Truths refer to intended values (e.g. **physical quantities under measurement**) or unexpected values (e.g. **true values + noises**) from the physical world.

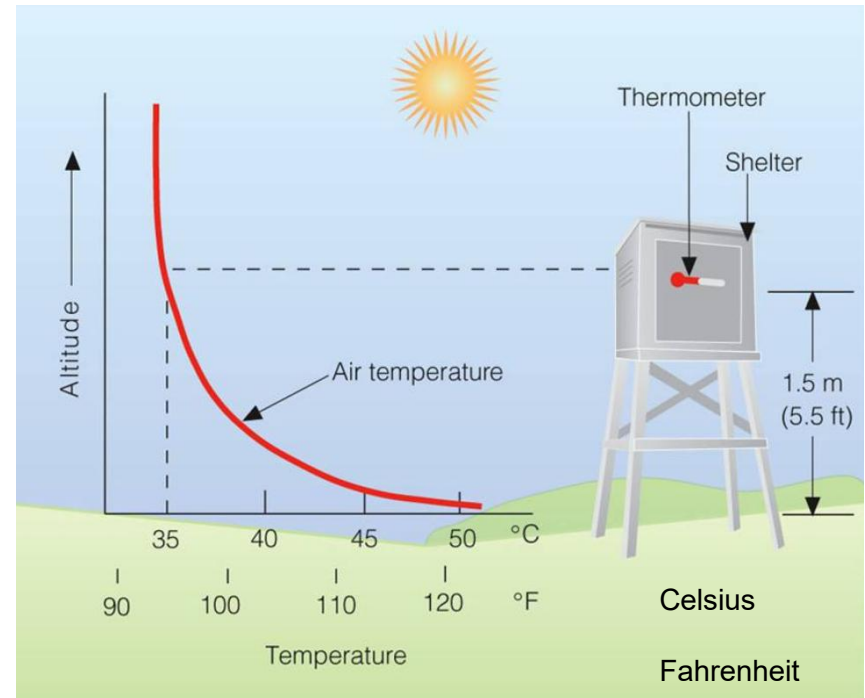
Example of Chance of Occurrence

- ▶ What is the chance for a car to appear from the opposite direction within the next 30 seconds?
- ▶ What is the chance for a car to appear behind my car within the next 15 seconds?



Example of Chance of Occurrence

- ▶ What is the chance for the air temperature to be at 35 degrees Celsius at 10h00 am?
- ▶ What is the chance for the air temperature to be at 26 degrees Celsius at 20h00 pm?



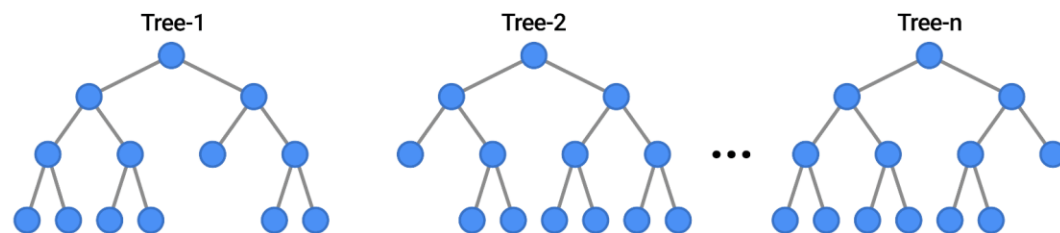
How to study randomness?

How to transform uncertainty into certainty?

Outline

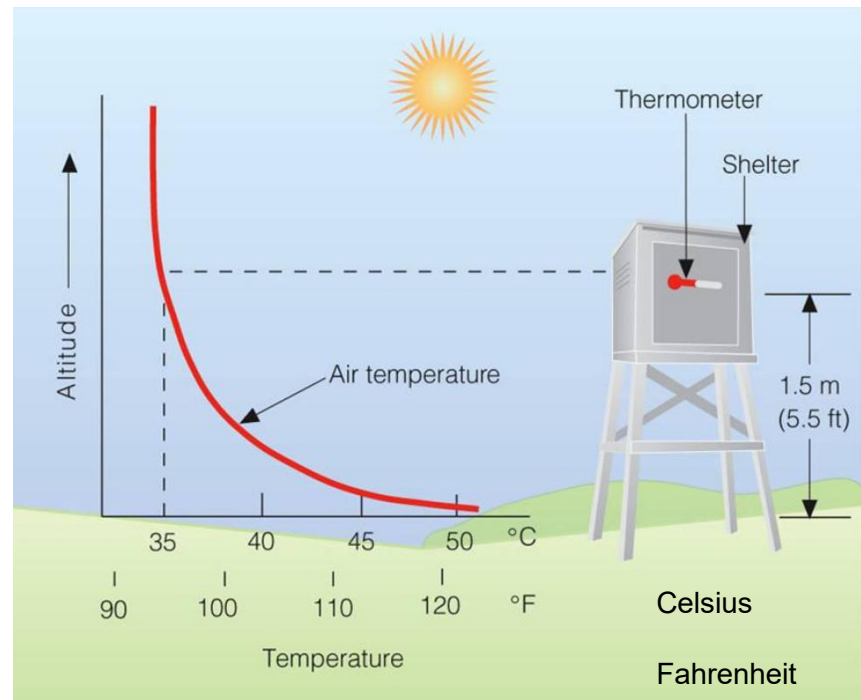
- ▶ Concept of Randomness
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EXAMPLES



Types of Values in the Physical World

- ▶ Analogue values
- ▶ Digital values
 - ▶ Logic values
 - ▶ Numeric values



Concept of Truth

- ▶ One value, or a combination of values, in the physical world is called a truth.

This is a truth

$$V_m = V_t + \Delta n$$

Measured Value

True Value

Variation Caused by Noise

Example

- ▶ A company is running a production line, which consists of **three processes**. Each process could be in one of the two states:
 - ▶ within control (**noise-level is within a limit**) and
 - ▶ out of control (**noise-level is beyond a limit**).
- ▶ What are the truths about the state of the operations in this production line?

Answer

- ▶ Let 1 represent the state of “within control”
- ▶ Let 0 represent the state of “out of control”.
- ▶ The set of truths about the eight states of operations is:

000	100
001	101
010	110
011	111

Process
1

Process
2

Process
3

Concept of Population

- ▶ A set of measurable or observable values in the physical world is called a **population** (or population in short).

Sub-Space for Truths

- ▶ Population = {Past, Present, Future}

Sub-Space for Samples

Concept of Sample Space

- ▶ For specific purpose or study, we may not consider the entire population.
- ▶ Instead, we can choose a sub-set in a population as the domain of interest.
- ▶ Such a sub-set in a population is called a **Sample Space**.

Concept of Truth Space

- ▶ A sub-set of truths collected from a population of truths is called a Truth Space (Note: Truth space includes sample space).

This is a truth

$$V_m = V_t + \Delta n$$

Measured Value

True Value

Variation Caused by Noise

Example

- ▶ A truth space of weather conditions could be {rainy, cloudy, sunny}.
- ▶ A truth space of product conditions could be {faulty, operational, good}.
- ▶ A truth space of academic outcomes could be {fail, pass, good, excellent}
- ▶ A truth space of traffic situations could be {jam, crowded, fluent}.
- ▶ A truth space of a sensor could be {20.1, 20.3, 19.7, 19.8, 20.2}

Mean Value of Truth Space

Sample Mean	Population Mean
$\bar{x} = \frac{\sum x}{n}$	$\mu = \frac{\sum x}{N}$

where $\sum X$ is sum of all data values

N is number of data items in population

n is number of data items in sample

Example

Sample	Observations in the Sample (n=4)				Sample Mean
1	25	50	80	85	60.0
2	25	50	80	90	61.3
3	25	50	80	100	63.8
4	25	50	85	90	62.5
5	25	50	85	100	65.0
6	25	50	90	100	66.3
7	25	80	85	90	70.0
8	25	80	85	100	72.5
9	25	80	90	100	73.8
10	25	85	90	100	75.0
11	50	80	85	90	76.3
12	50	80	85	100	78.8
13	50	80	90	100	80.0
14	50	85	90	100	81.3
15	80	85	90	100	88.8

Standard Deviation of Truth Space

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

σ = standard deviation

\sum = sum of

x = each value in the data set

\bar{x} = mean of all values in the data set

n = number of value in the data set

Example

- ▶ A sensor outputs a series of numbers such as {3, 12, 9, 4, 2}. Then, we can compute:

$$\bar{X} = \frac{(3 + 12 + 9 + 4 + 2)}{5} = 6$$

$$s = \sqrt{\frac{\sum_{j=1}^N (X_j - \bar{X})^2}{N}}$$

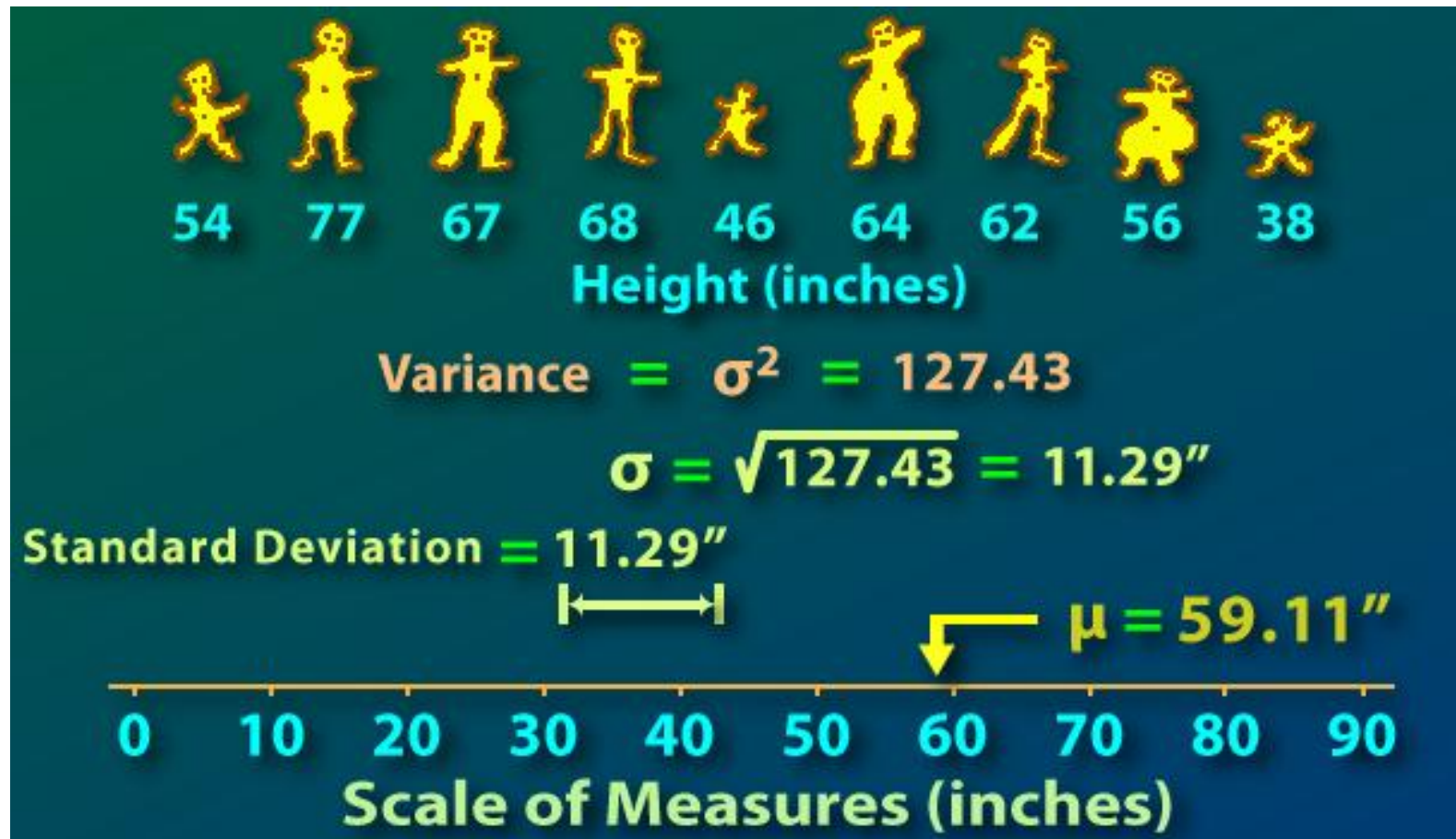
$$s = \sqrt{\frac{(3-6)^2 + (12-6)^2 + (9-6)^2 + (4-6)^2 + (2-6)^2}{5}}$$

$$s = \sqrt{\frac{(9+36+9+4+16)}{5}}$$

$$s = \sqrt{\frac{74}{5}} \approx 3.847$$

Example

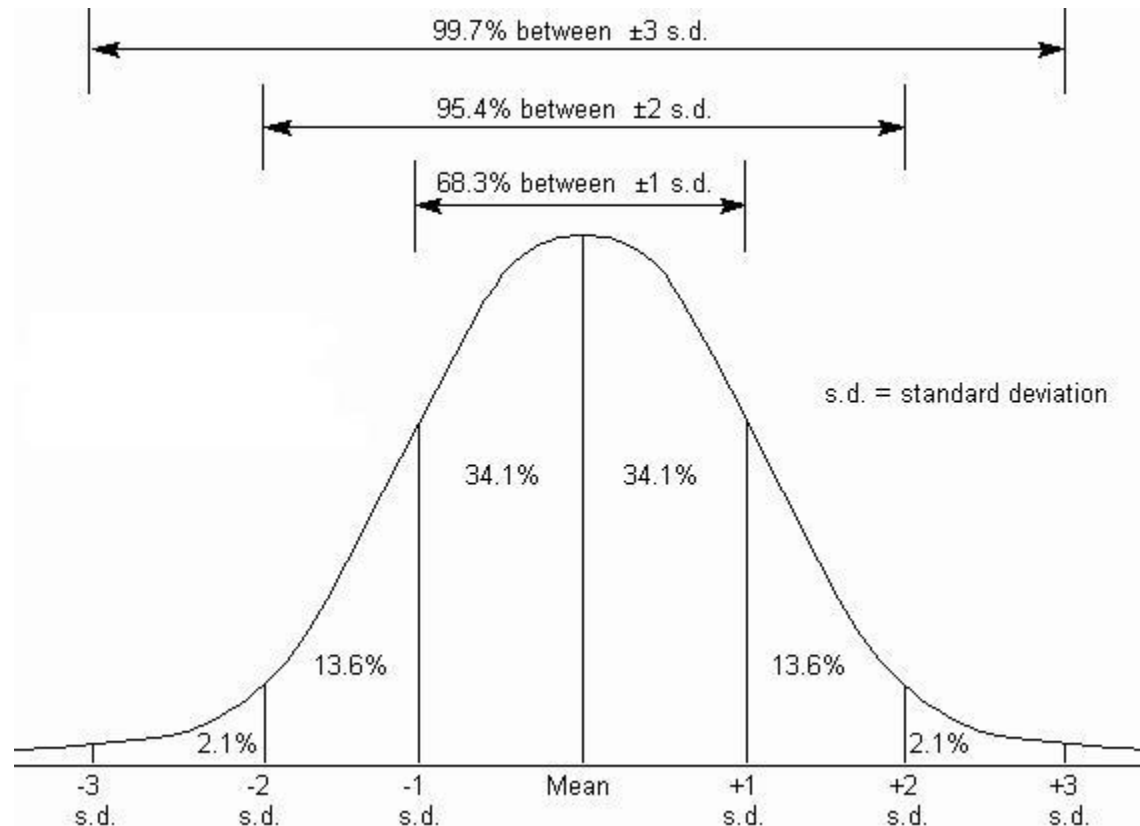
- ▶ The heights of a family's members are {54, 77, 67, 68, 46, 64, 62, 56, 38} (inches). Then, we can compute:



Concept of Statistics

- ▶ Statistics refer to the calculated values from all the data of truth space.
- ▶ **Statistics** indicate the **properties** of truth space.

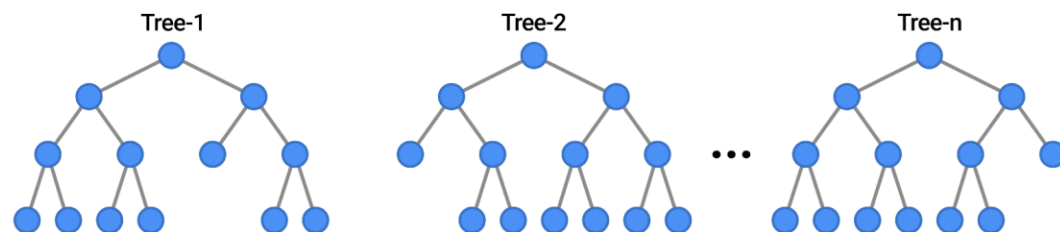
Use of Mean and Standard Deviation



Outline

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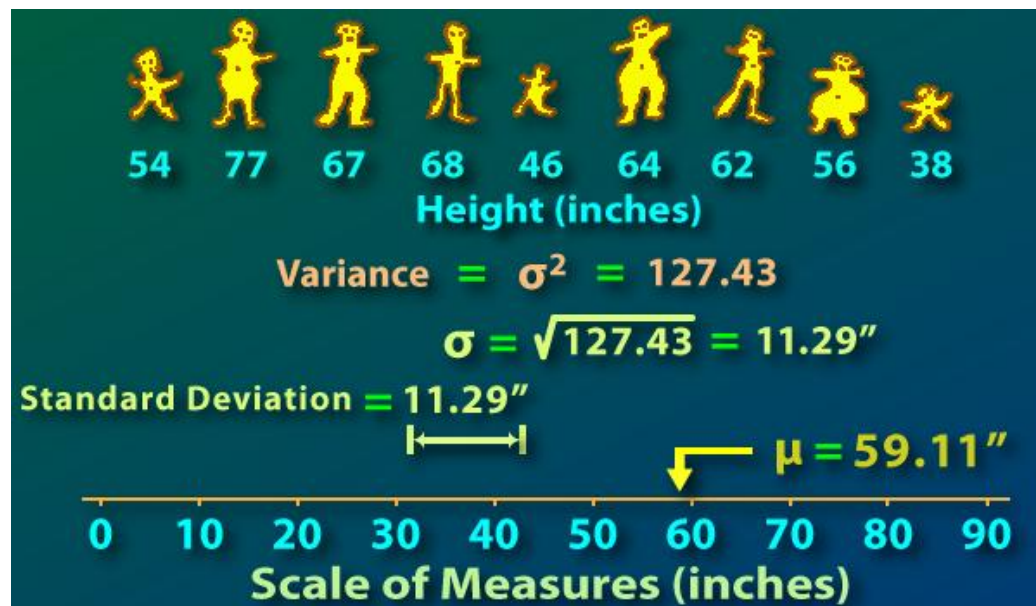
EXAMPLES



A truth is a set

It is because

- ▶ A truth could be a combination of values from the physical world.



Example of Truth Sets

- ▶ In a production line of assembling connectors, there is a station for inspecting assembled connectors. The inspection station takes **three connectors** for inspection each time. The outcomes of an inspection are either good or defective.
- ▶ What is **truth set A** that at least one connector is good?
- ▶ What is **truth set B** that only two connectors are defective?

Answer

- ▶ Let 1 represent the case of a good connector
- ▶ Let 0 represent the case of a defective connector
- ▶ Truth set A, that at least one connector is good, is
 - ▶ {111, 110, 101, 100, 011, 010, 001}
- ▶ Truth set B, that only two connectors are defective, is
 - ▶ {001, 010, 100}

Connector
1

Connector
2

Connector
3

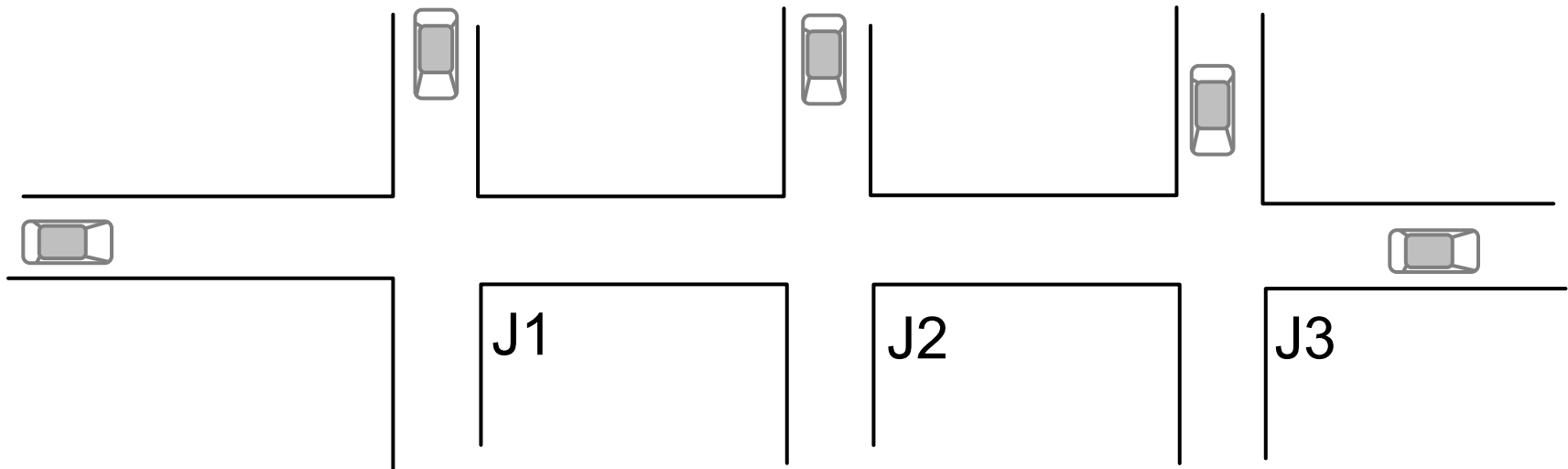
Operation 1: Intersection of Truths

- ▶ Two (or more) sets of truths share common truths.

$$C = A \cap B$$

Example

- ▶ You are driving a car and enter a boulevard which has a series of three junctions. At each junction, the traffic light can be in one of the two states such as Working or Faulty. What is truth A that at least one traffic light is in working state? What is truth B that at least one traffic light is in faulty state? What is the intersection of truth A and truth B?



Answer

- ▶ Let W represent traffic light in working state, and F represent traffic light in faulty state.
- ▶ Truth A = $\{WFF, WFW, WWF, WWW, FWF, FFW, FWW\}$
- ▶ Truth B = $\{FWW, FWF, FFW, FFF, WFW, WWR, WFF\}$
- ▶ Intersection of truth A and truth B is:
 - ▶ $\{WFF, WFW, WWF, FWF, FFW, FWW\}$

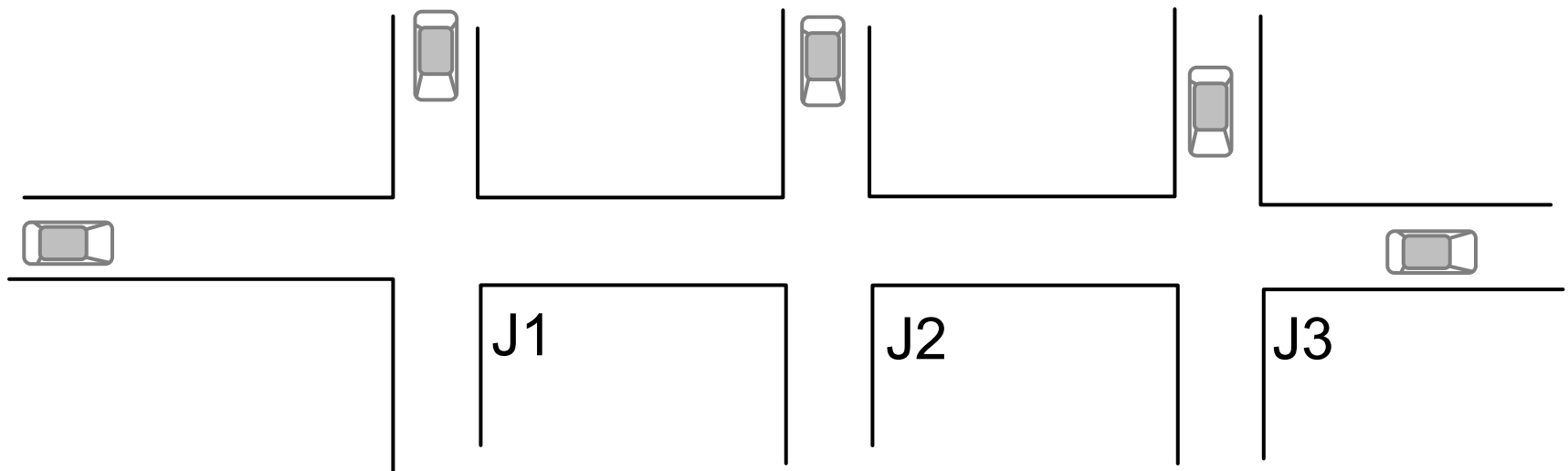
Operation 2: Union of Truths

- ▶ Two (or more) sets of truths are combined into one set of truths.

$$C = A \cup B$$

Example

- ▶ Along a boulevard, there is a series of three junctions. Each traffic light can be in either working state or faulty state. What is truth A that at least junction 1's traffic light is in working state? What is truth B that at least junction 2's traffic light is in working state? What is the union of A and B?



Answer

- ▶ Let W represent traffic light in working state, and F represent traffic light in faulty state.
- ▶ Truth A = $\{WFF, WFW, WWF, WWW\}$
- ▶ Truth B = $\{FWF, WWF, FWW, WWW\}$
- ▶ Union of A and B is
 - ▶ $\{WFF, WFW, WWF, WWW, FWF, WWF, FWW\}$

Special Case: Exclusive Truth Sets

- ▶ When two sets of truths do not have any intersection in a same truth space, these two sets of truths are mutually exclusive.
- ▶ Exclusive truth sets within a same truth space cannot occur at the same time.
- ▶ Exclusive truth sets within a same truth space are competing truths.

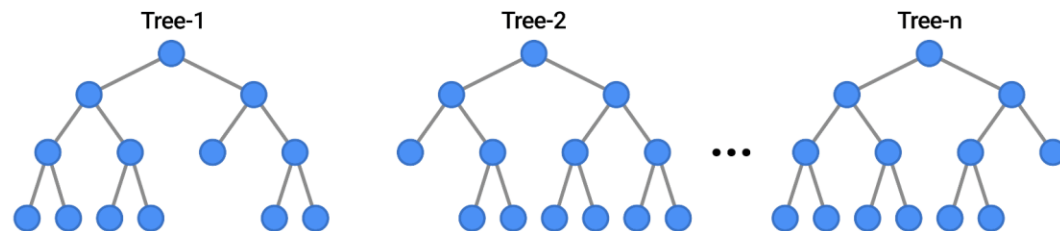
Example

- ▶ Exclusive truth sets of product conditions:
 - ▶ {good} verse {defective}
- ▶ Exclusive truth sets of material states:
 - ▶ {solid}, {liquid}, and {gas}
- ▶ Exclusive truth sets of leg actions:
 - ▶ {push}, {swing}, {support}, and {carry}
- ▶ Exclusive truth sets of gender:
 - ▶ {male} verse {female}

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EXAMPLES



Truths can compete for occurrence

For example

- ▶ The truth of *being rainy* faces the competitors such as *being sunny*, and *being cloudy*.
- ▶ The truth of *being good* faces the competitor of *being defective*.
- ▶ The truth of *being red traffic light* faces the competitors such as *being yellow traffic light* and *being green traffic light*.
- ▶ The truth of *being in working condition* faces the competitor of *being in faulty condition*.

Can we truthfully predict a winner in a competition?

Answer:

- ▶ No, one cannot truthfully predict the winner in a competition.
- ▶ However, one can evaluate the chance for an entry to be the winner (i.e. **chance of occurrence**) in a competition.

How to study the chance of occurrence of a truth which has other competing truths?

Answer:

► To use the theory of probability.

Definition of Probability

- ▶ Probability is a percentage which **predicts** the chance of occurrence of a truth in the presence of all the other competing truths.

Properties of Probability

- ▶ Probability is about prediction.
- ▶ Probability is about predicted chance of occurrence of a truth.

Computation of Probability

- ▶ If T represents a truth, the probability of T is:

$$P(T) = \frac{\text{Occurrence of T}}{\text{Total Occurrence}}$$

Rule 1 of Probability

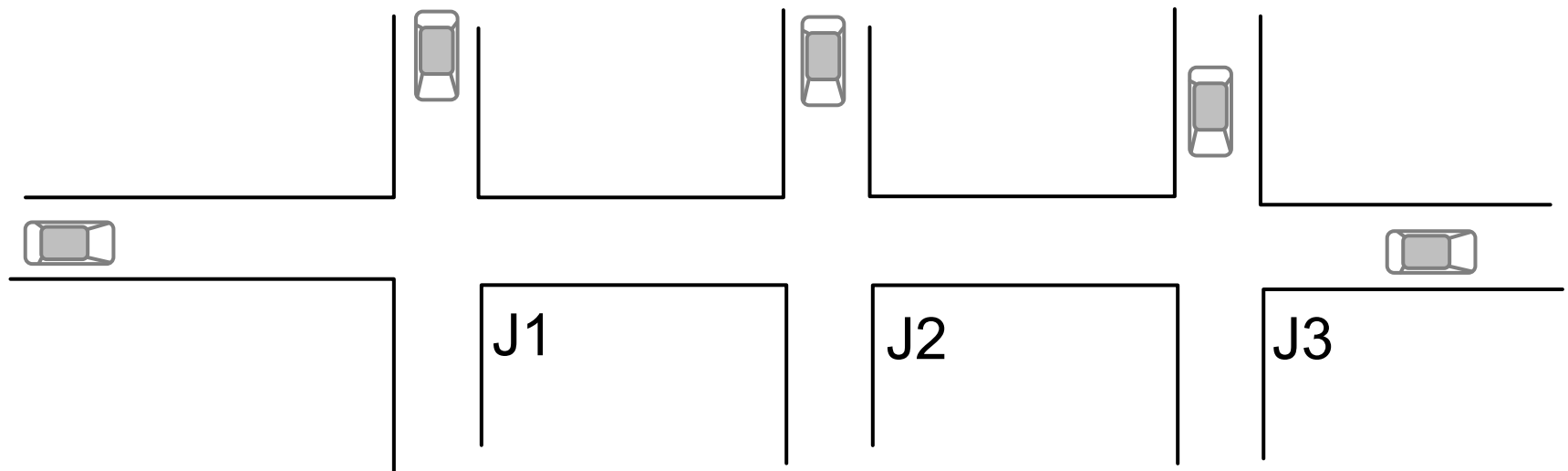
- ▶ If a truth does not have other competing truth, its probability is equal 1.

Example:

- ▶ Truth A is that the Earth is rotating about the Sun. Then, $P(A)$ is equal to 1.

Example

- ▶ Your smart car is moving along a boulevard which has a series of three junctions. Each traffic light can be changed to either red or green. Assume that three traffic lights are independent and that they have equal chance to be in red or green. What is the probability for three traffic lights to be in red together?



Answer

- ▶ Let R represent red traffic light and G represent green traffic light.
- ▶ Truth Space = {GGG, GGR, GRG, GRR, RGG, RGR, RRG, RRR}

$$P(\text{Truth Space}) = 1$$

- ▶ Probability of RRR is:

$$P(RRR) = \frac{1}{8}$$

Rule 2 of Probability

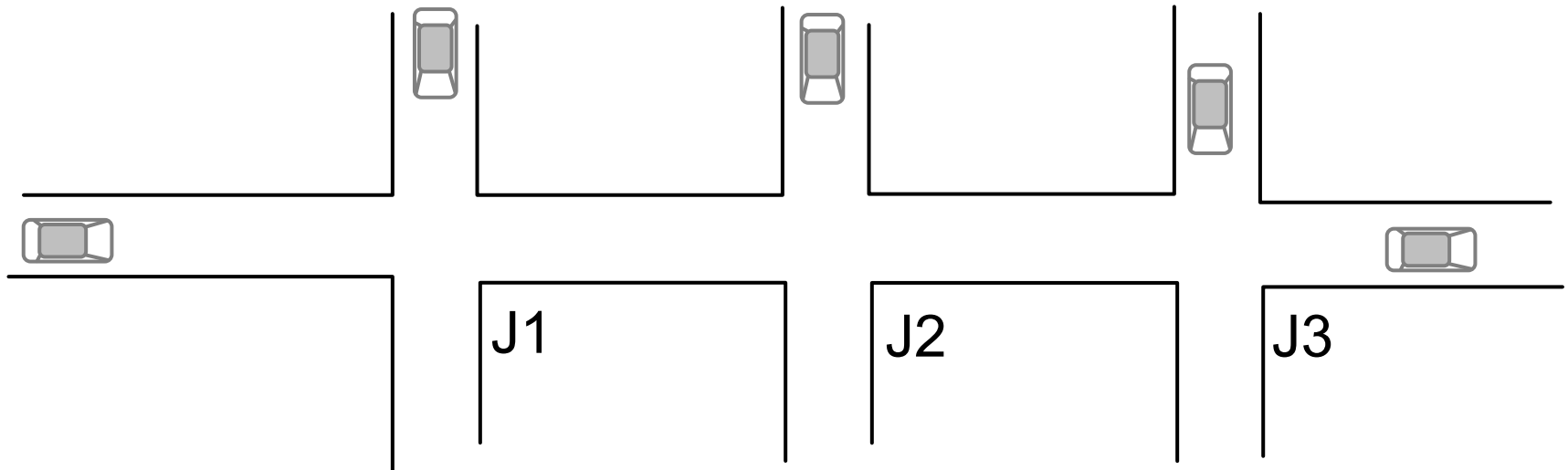
- ▶ If set A contains multiple truths, then $P(A)$ is the sum of the probability of each truth.

$$T = \{e_1, e_2, \dots, e_n\}$$

$$P(T) = P(e_1) + P(e_2) + \dots + P(e_n)$$

Example

- ▶ You drive a smart car along a boulevard which has a series of three junctions. Each traffic light can be changed to either red or green. What is the probability of truth A that at least two traffic lights are in green?



Answer

- ▶ Let R represent red traffic light and G represent green traffic light.
- ▶ Truth A = {GGR, GRG, RGG, GGG}
- ▶ Probability of A is:

$$\begin{aligned} P(A) &= P(GGR) + P(GRG) + P(RGG) + P(GGG) \\ &= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2} \end{aligned}$$

Rule 3 of Probability

- ▶ If truth set B is the complement of truth set A, then the sum of probabilities of A and B is equal to one. It means that it is 100% certain that one of them will occur.

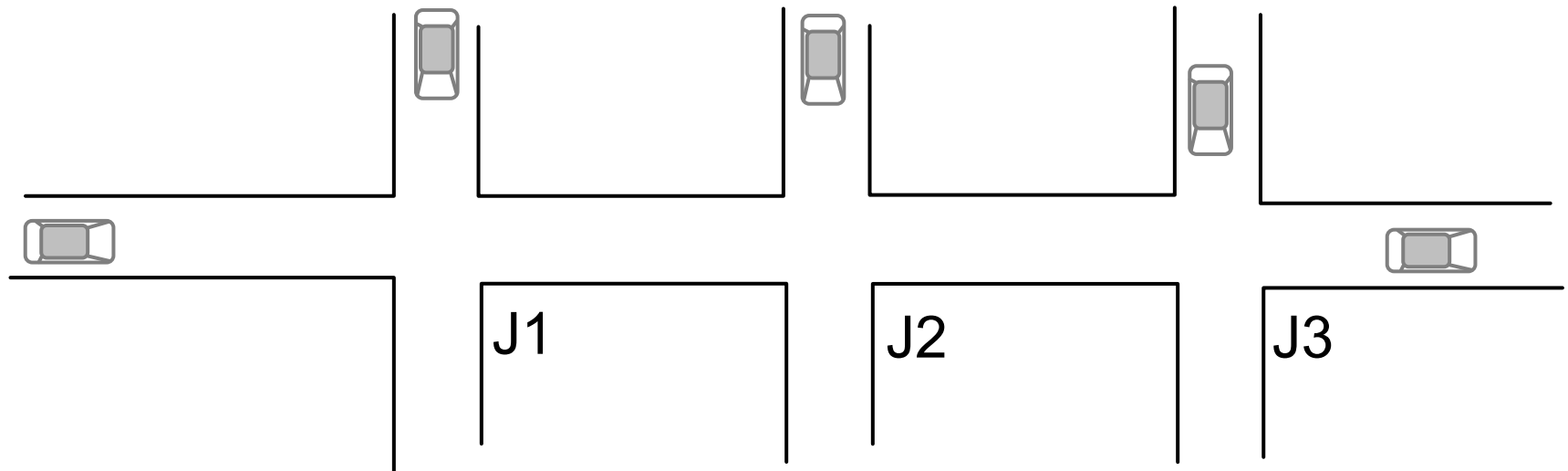
$$B = \neg A$$

$$P(A) + P(B) = 1$$

$$P(\neg A) = 1 - P(A)$$

Example

- ▶ You drive a smart car along a boulevard which has a series of three junctions. Each traffic light can be changed to either red or green. Truth A is that at least two traffic lights are in green. What is the complement of truth A?



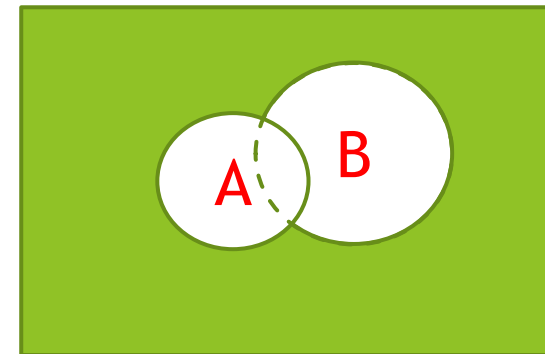
Answer

- ▶ Let R represent red traffic light and G represent green traffic light.
- ▶ Truth A = {GGR, GRG, RGG, GGG}
- ▶ Complement of A is:
 - ▶ {GRR, RGR, RRG, RRR}

Rule 4 of Probability

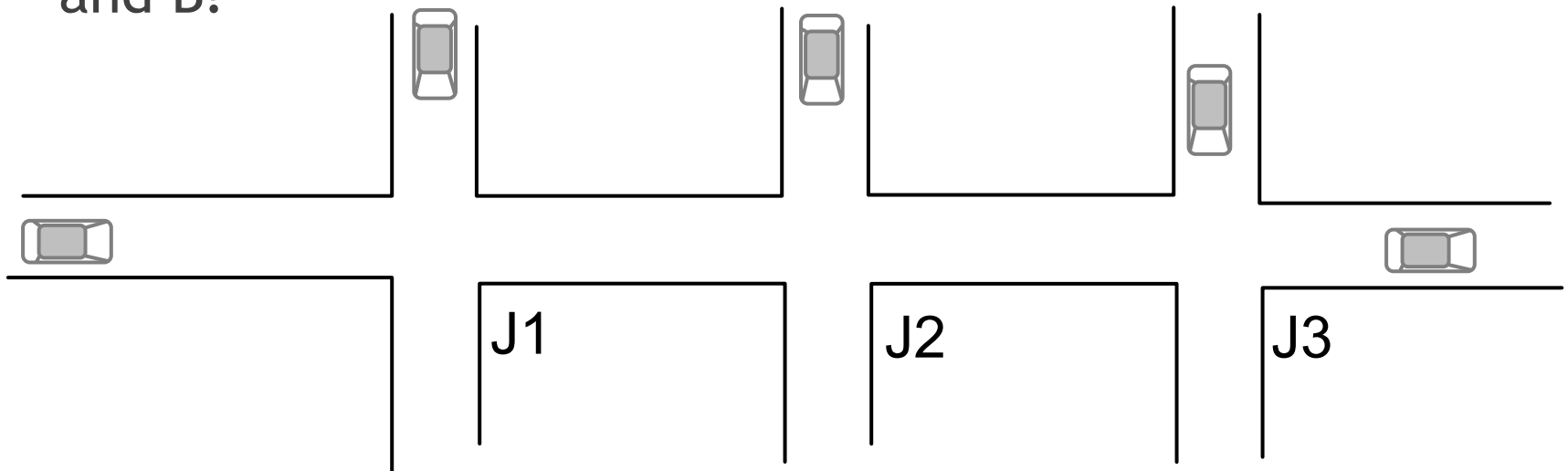
- ▶ Truth A and truth B are not mutually exclusive.
- ▶ The probability of their union is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Example

- ▶ You drive a smart along a boulevard which has a series of three junctions. Each traffic light can be changed to either red or green. Truth A is that at least junction 1's traffic light is in green. Truth B is that at least junction 2's traffic light is in green. What is the probability of the union of A and B?



Answer

- ▶ Let R represent red traffic light, and G represent green traffic light.
- ▶ Truth A = {GRR, GRG, GGR, GGG}
- ▶ Truth B = {RGR, GGR, RGG, GGG}
- ▶ Union of A and B is: {GRR, GRG, GGR, GGG, RGR, RGG}
- ▶ Intersection of A and B is: {GGR, GGG}
- ▶ The probability of their union is:

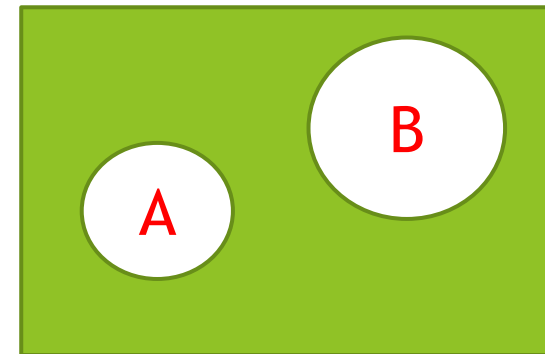
$$\begin{aligned}
 P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
 &= P(GRR) + P(GRG) + P(GGR) + P(GGG) + P(RGR) + P(RGG)
 \end{aligned}$$

Rule 5 of Probability

- ▶ Truth A and truth B are mutually exclusive. The probability of their union is:

$$P(A \cup B) = P(A) + P(B)$$

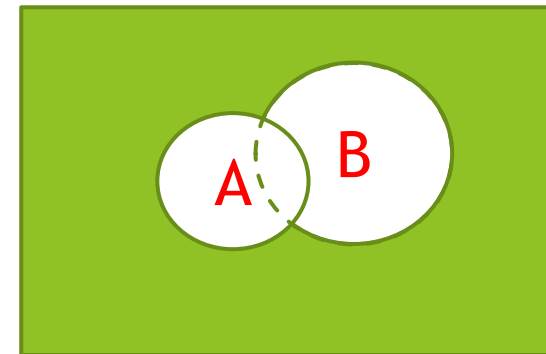
$$A \cap B = \text{Null}$$



Rule 6 of Probability

- ▶ Truth A and truth B are not mutually exclusive. The probability of their intersection is:

$$P(A \cap B) = P(A)P(B | A)$$



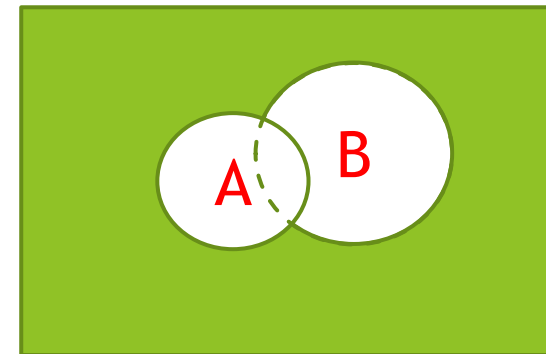
Proof

$$P(A \cap B) = \frac{\text{Occurrence of A and B}}{\text{Total Occurrence}}$$

$$P(A \cap B) = \frac{\text{Occurrence of A and B}}{\text{Total Occurrence}} \times \frac{\text{Occurrence of A}}{\text{Occurrence of A}}$$

$$P(A \cap B) = P(A) \times \frac{\text{Occurrence of A and B}}{\text{Occurrence of A}}$$

$$P(A \cap B) = P(A) \times P(B | A)$$

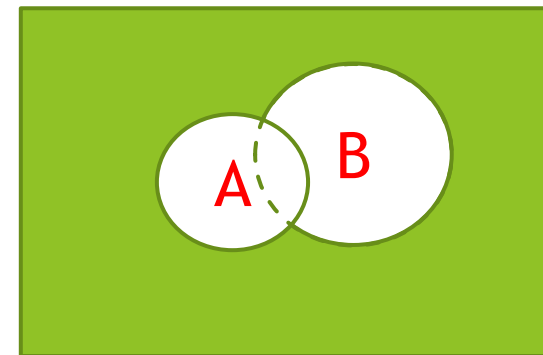


Definition of Conditional Probability

- ▶ If truth A and truth B are related, the probability of B given A (or A given B) is called a conditional probability.

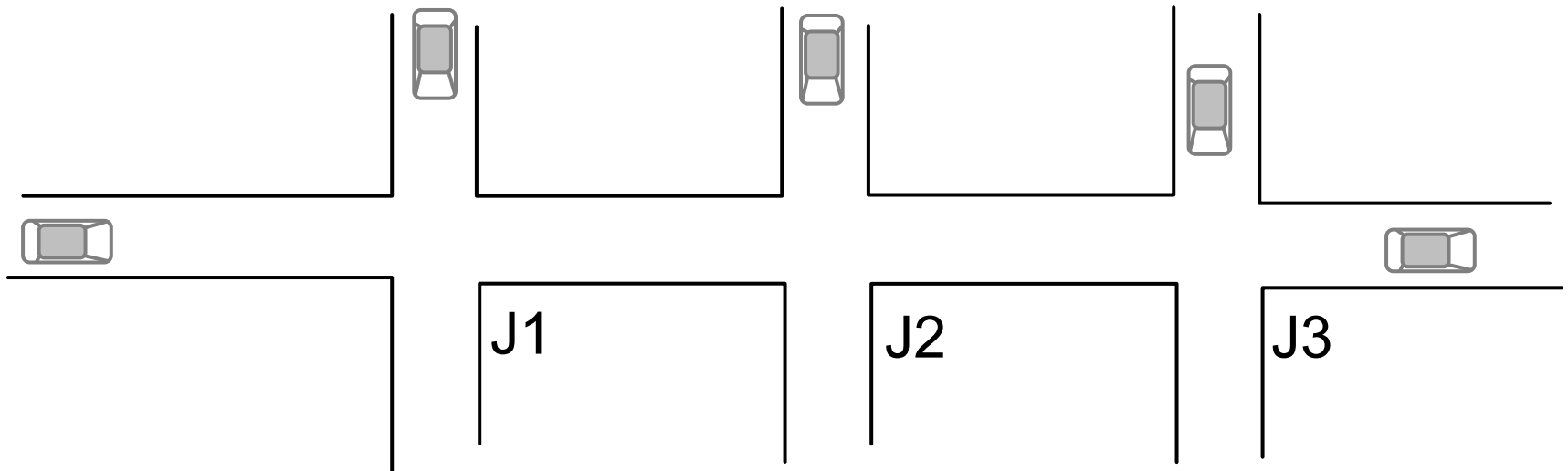
$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B)$$

$$P(B | A) = \frac{P(A \cap B)}{P(A)}$$



Example

- ▶ Along a boulevard, there is a series of three junctions. Each traffic light can be changed to either red or green. Truth A is that at least junction 1's traffic light is in green. Truth B is that at least junction 2's traffic light is in green. What is the probability of the intersection of A and B?



Answer

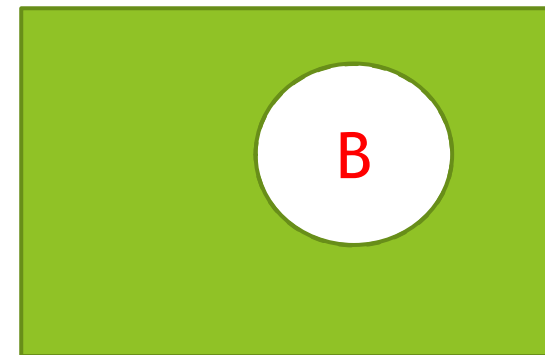
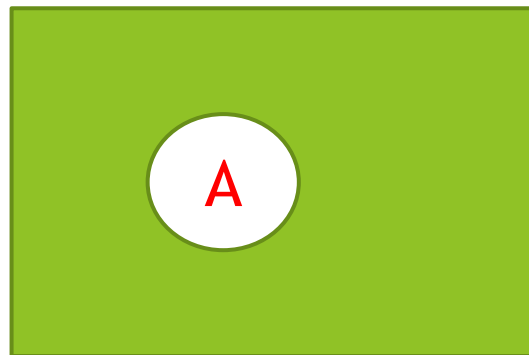
- ▶ Let R represent red traffic light, and G represent green traffic light.
- ▶ Truth A = {GRR, GRG, GGR, GGG}
- ▶ Truth B = {RGR, GGR, RGG, GGG}
- ▶ Union of A and B is: {GRR, GRG, GGR, GGG, RGR, RGG}
- ▶ Intersection of A and B is: {GGR, GGG}
- ▶ The probability of their intersection is:

$$\begin{aligned}P(A \cap B) &= P(A)P(B | A) \\ &= \frac{4}{8} \bullet \frac{2}{4} = \frac{2}{8}\end{aligned}$$

Rule 7 of Probability

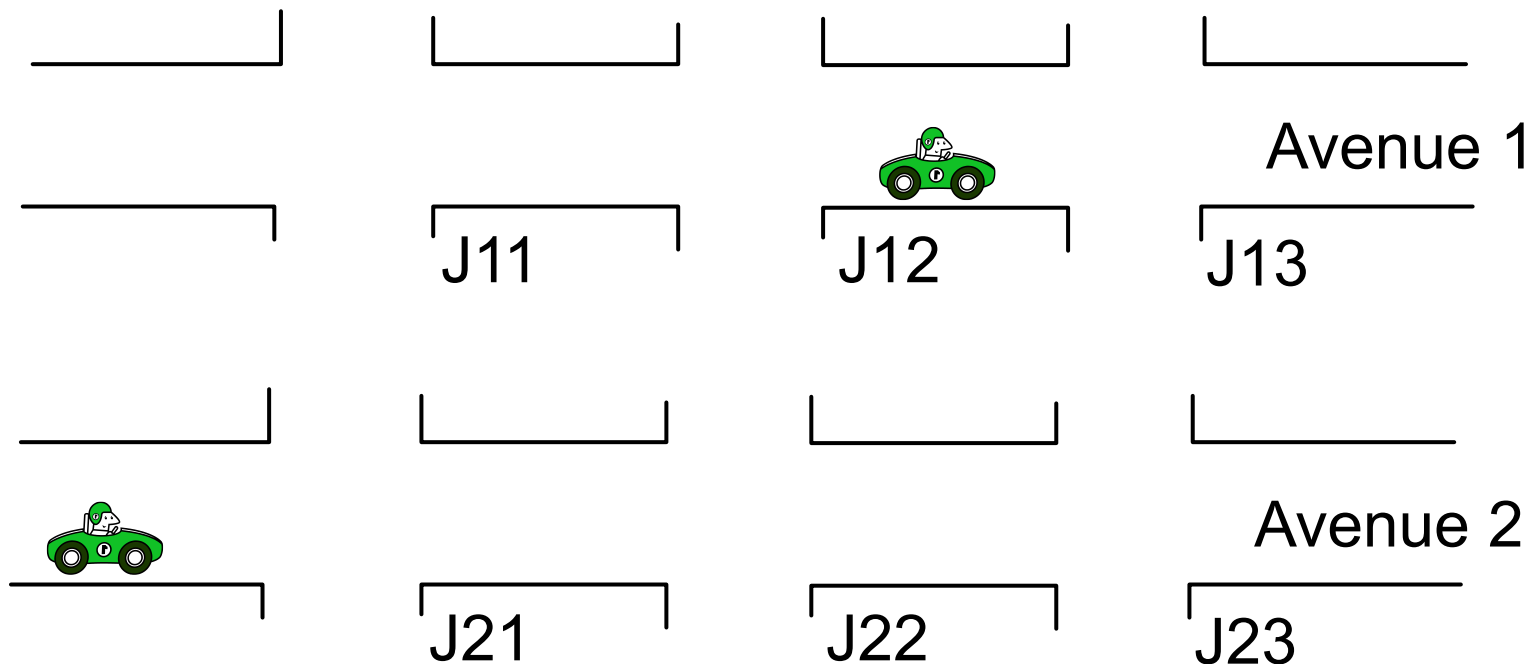
- ▶ Truth A and truth B are from two different truth spaces. Then, A and B are independent. The probability of their intersection is:

$$P(A \cap B) = P(A)P(B)$$



Example

- ▶ We have two parallel avenues in a city, each of which has three traffic lights that are independently operated. Truth A is that at least two traffic lights in avenue 1 are in green. Truth B is that at least two traffic lights in avenue 2 are in red. What is the probability of the intersection of A and B?



Answer

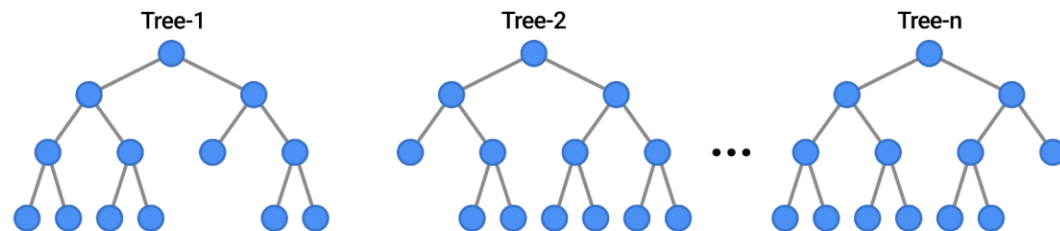
- ▶ Let R represent red traffic light, and G represent green traffic light.
- ▶ Truth A = {GGR, GRG, RGG, GGG}
- ▶ Truth B = {RRG, RGR, GRR, RRR}
- ▶ A and B are independent because they are from different truth spaces.
- ▶ The probability of their intersection is:

$$\begin{aligned}P(A \cap B) &= P(A)P(B) \\ &= \frac{4}{8} \bullet \frac{4}{8} = \frac{1}{4}\end{aligned}$$

Outline

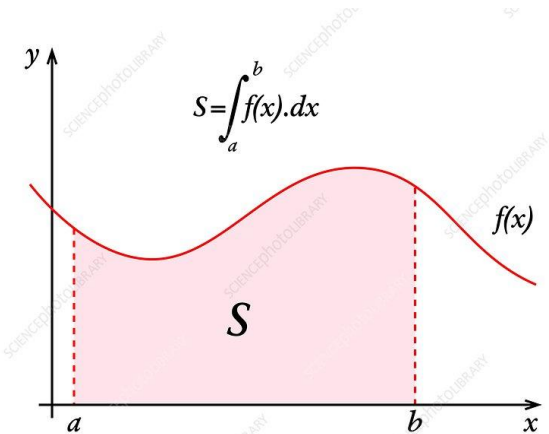
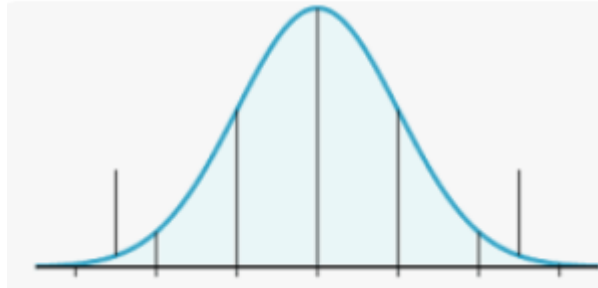
- ▶ Concept of Randomness
- ▶ Statistics of Truth
- ▶ Operations of Truth
- ▶ Probability of Truth
- ▶ Determination of Probability
- ▶ Prediction Using Bayes' Theorem

EXAMPLES



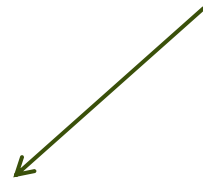
How to determine probability?

Answer:



- ▶ Use of recorded data in the past
- ▶ Use of experiences
- ▶ Use of probability mass (i.e., **distribution**) functions
- ▶ Use of probability (**distribution**) density functions

Misleading Wording



Determination of Probability from Recorded Data

- ▶ We have a set of exclusive truths. The probability of one exclusive truth is the ratio between the occurrence of the truth and the total occurrence of all the exclusive truths in the set.

$$P(T_i) = \frac{\text{Occurrence of } T_i}{\text{Total Occurrence}}$$

Example

- ▶ The past year records show that Beijing had 100 of rainy days, 150 days of sunny days and 115 days of cloudy days in 2012. What is the probability for tomorrow to be a sunny day?

Answer:

$$P(\textit{Sunny}) = \frac{150}{365} = 41.1\%$$

Determination of Probability from Experiences

Example:

- ▶ In a production line of assembling a product, there are five types of defects. According to experiences in the past years, there is an equal chance for each of these five defects to occur. If each defect is being measured by a sensor which will output 1 when the defect occurs, what is the probability for sensor No. 3 to output 1 in the next minute?

Answer:

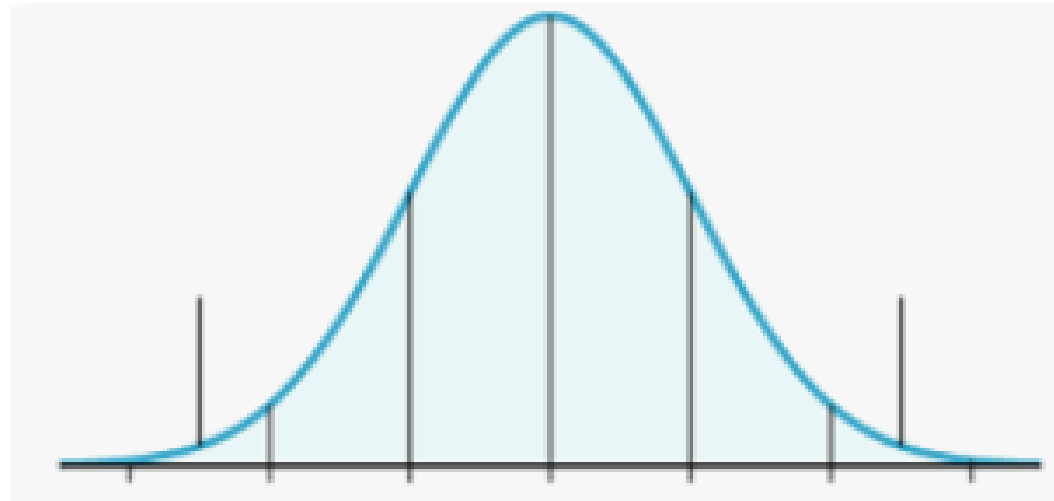
$$P(T_3) = \frac{1}{5} = 20.0\%$$

Determination of Probability from Mass (i.e., Distribution) Functions

If $f(x_i)$ is a probability mass function, then :

$$f(x_i) \geq 0$$

$$\sum_{i=1}^n f(x_i) = 1$$



Example

- ▶ Let T denote the number of semiconductor wafers that need to be analyzed in order to detect a large particle of contamination. Assume that the probability that a wafer contains a large particle is 0.01, and that the wafers are independent. Determine the probability mass function of T .

Answer

- ▶ Let 'b' indicate that the a large particle is “being present”, and 'a' indicate that a large particle is “being absent”. Wafers are independent.
- ▶ Then, the truth space of T consists of independent sub-truth
- ▶ Hence, T is:
 - ▶ $T = \{b, ab, aab, aaaab, \dots\}$
- ▶ The probability mass function of T will be:

$$P(T = 1) = P(b) = 0.01$$

$$P(T = 2) = P(ab) = P(a)P(b) = 0.99 \bullet 0.01$$

$$P(T = n) = P(a^{n-1}b) = P^{n-1}(a)P(b) = 0.99^{n-1} \bullet 0.01$$

Typical Probability Mass Functions

Uniform Function

$$f(x_i) = \frac{1}{n}, i = 1, 2, \dots, n$$

Binomial Function

$$f(k) = \binom{n}{k} p^k (1-p)^{n-k}, k = 0, 1, 2, \dots, n.$$

Poisson Function

$$f(k) = \frac{e^{-\lambda} \lambda^k}{k!}, k = 0, 1, 2, \dots, n$$

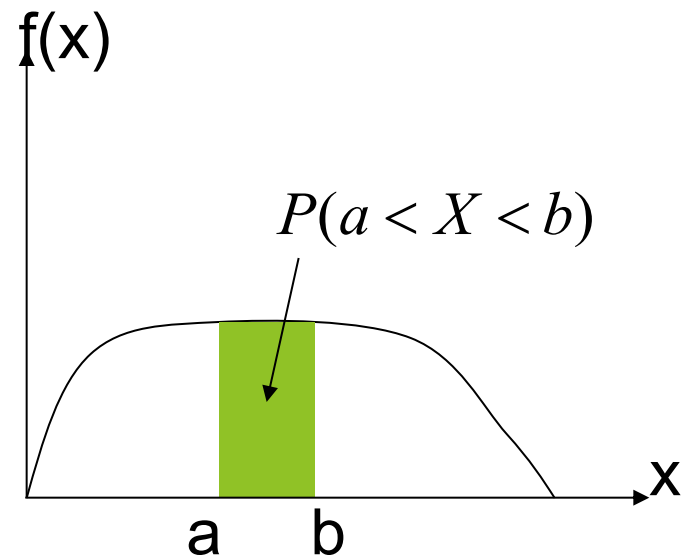
Determination of Probability from (Distribution) Density Functions

$f(x)$ is a probability density function if it satisfies :

1. $f(x) \geq 0$

2. $\int_{-\infty}^{\infty} f(x) dx = 1$

3. $P(a \leq X \leq b) = \int_a^b f(x) dx$



Exercise

Let the truth X represent the diameter of a hole drilled in a metal component. The target diameter is 12.5 millimeters. It was found that most random disturbances to the process will result in obtaining larger diameters. And, historical data show that the probability of X can be modeled by a probability density function :

$$f(x) = 20e^{-20(x-12.5)}, x \geq 12.5$$

If a part with a diameter larger than 12.60 millimeters is scrapped, what portion of parts will be scrapped?

Answer

$$f(x) = 20e^{-20(x-12.5)}, x \geq 12.5$$

$$P(X > 12.6) = \int_{12.6}^{\infty} 20e^{-20(x-12.5)} dx = -e^{-20(x-12.5)} \Big|_{12.6}^{\infty} = 0.135$$

So, the portion of 13.5% of parts will be scrapped.

Probability Density Function of Uniform Distribution

Truth X represent the continuous variable assigned to an experiment or trials with competing outcomes.

If the probability density function of X can be described by :

$$f(x) = \frac{1}{b-a}, a \leq x \leq b$$

then X has a uniform distribution.

Probability Density Function of Normal Distribution

Truth X represent the continuous variable assigned to an experiment or trials with competing outcomes.

If the probability density function of X can be described as :

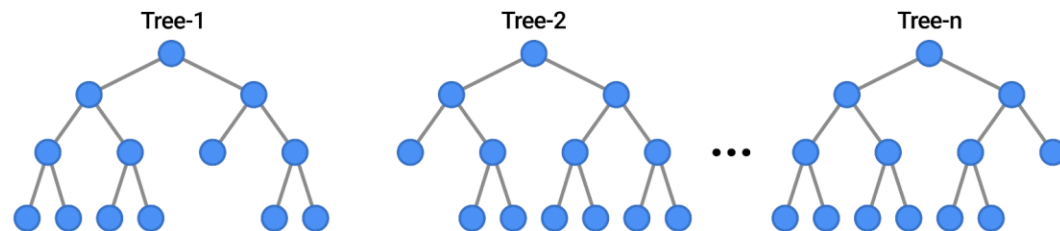
$$f(x) = N(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

then X has a normal distribution with parameters μ and σ where the mean of X is μ and the standard deviation of X is σ .

Outline

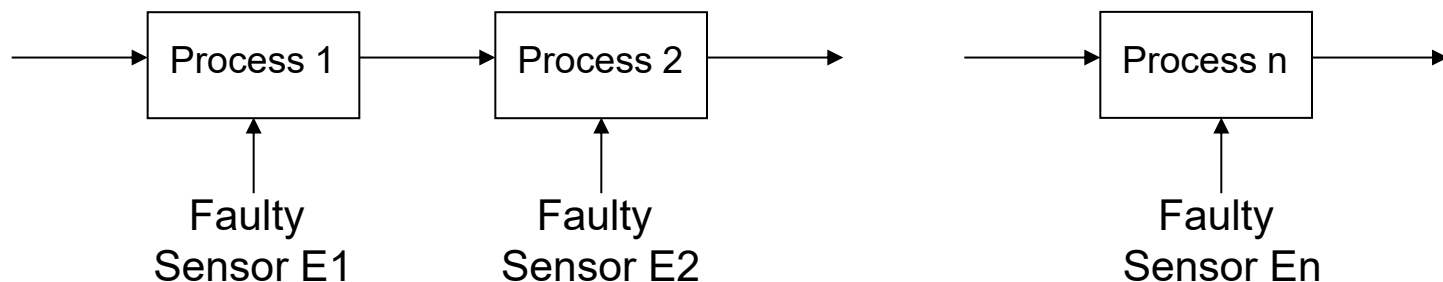
- ▶ Concept of Randomness
- ▶ Statistics of Truth
- ▶ Operations of Truth
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EXAMPLES



Example

- ▶ A production line has n processes which are independent. If none of the processes is faulty, the production line is in normal condition. Assume that only one process could be faulty at one time. When there is one faulty process, the production line will be under repair condition. Let truth B represent the fact that the production line is in repair condition. Let truth E_i represent the fact that process i 's sensor is faulty. What is the probability of B ? What is the probability of E_1 when B occurs?



Answer

- ▶ Truths of causes: $\{E_1, E_2, E_3, \dots, E_n\}$
- ▶ Truth of effect: $\{B\}$
- ▶ Probability of B:

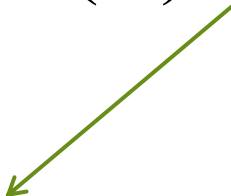
$$P(B) = P(B \text{ due to } E_1) + P(B \text{ due to } E_2) + \dots + P(B \text{ due to } E_n)$$

$$P(B) = P(E_1 \cap B) + P(E_2 \cap B) + \dots + P(E_n \cap B)$$

Answer (continued)

- Probability of intersection between B and E1:

$$P(E_1 \cap B) = P(E_1 | B)P(B) = P(B | E_1)P(E_1)$$


$$P(E_1 | B) = \frac{P(B | E_1)P(E_1)}{P(B)}$$



This is the probability of E1 when B occurs.

Answer (continued)

$$P(E_1 | B) = \frac{P(B | E_1)P(E_1)}{P(B)}$$



$$P(E_1 | B) = \frac{P(B | E_1)P(E_1)}{P(E_1 \cap B) + \dots + P(E_n \cap B)}$$



$$P(E_1 | B) = \frac{P(B | E_1)P(E_1)}{P(B | E_1)P(E_1) + \dots + P(B | E_n)P(E_n)}$$

Bayes' Theorem

Truth of effect A is related to a set of truths of causes $\{E_i, i = 1, 2, \dots, n\}$.
The truths of causes are mutually exclusive.

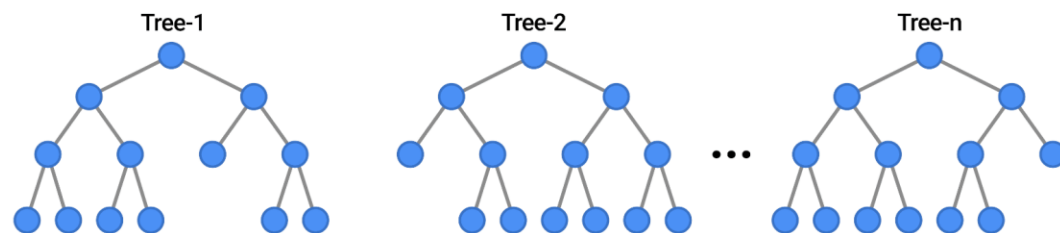
When A occurs, the probability for us to conclude that the cause is E_i can be computed by :

$$P(E_i | A) = \frac{P(A | E_i)P(E_i)}{P(A | E_1)P(E_1) + P(A | E_2)P(E_2) + \dots + P(A | E_n)P(E_n)}$$

Summary

- ▶ Concept of Randomness
- ▶ Statistics of Truth
- ▶ Operations of Truth
- ▶ Probability of Truth
- ▶ Determination of Probability
- ▶ Prediction Using Bayes' Theorem

EXAMPLES



Outline of Module 1

- ▶ Lecture 1:
 - ▶ Basics of Physical World
- ▶ Lecture 2:
 - ▶ Randomness of Physical World
- ▶ **Lecture 3:**
 - ▶ **Basics of Conceptual Worlds**
- ▶ Lecture 4:
 - ▶ Fuzziness of Conceptual Worlds





**NANYANG
TECHNOLOGICAL
UNIVERSITY**

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 1 Lecture 3

MA4822

Basics of Conceptual Worlds

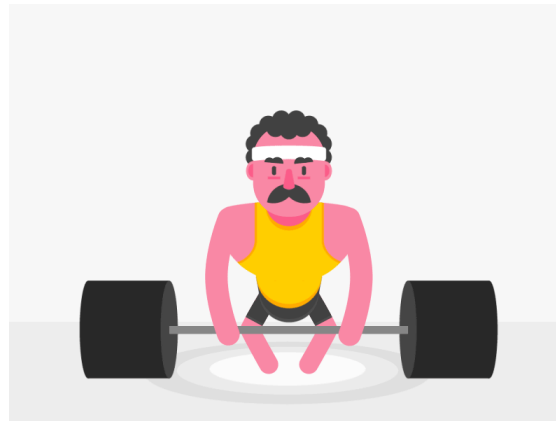
Xie Ming, PhD (France)

mmxie@ntu.edu.sg

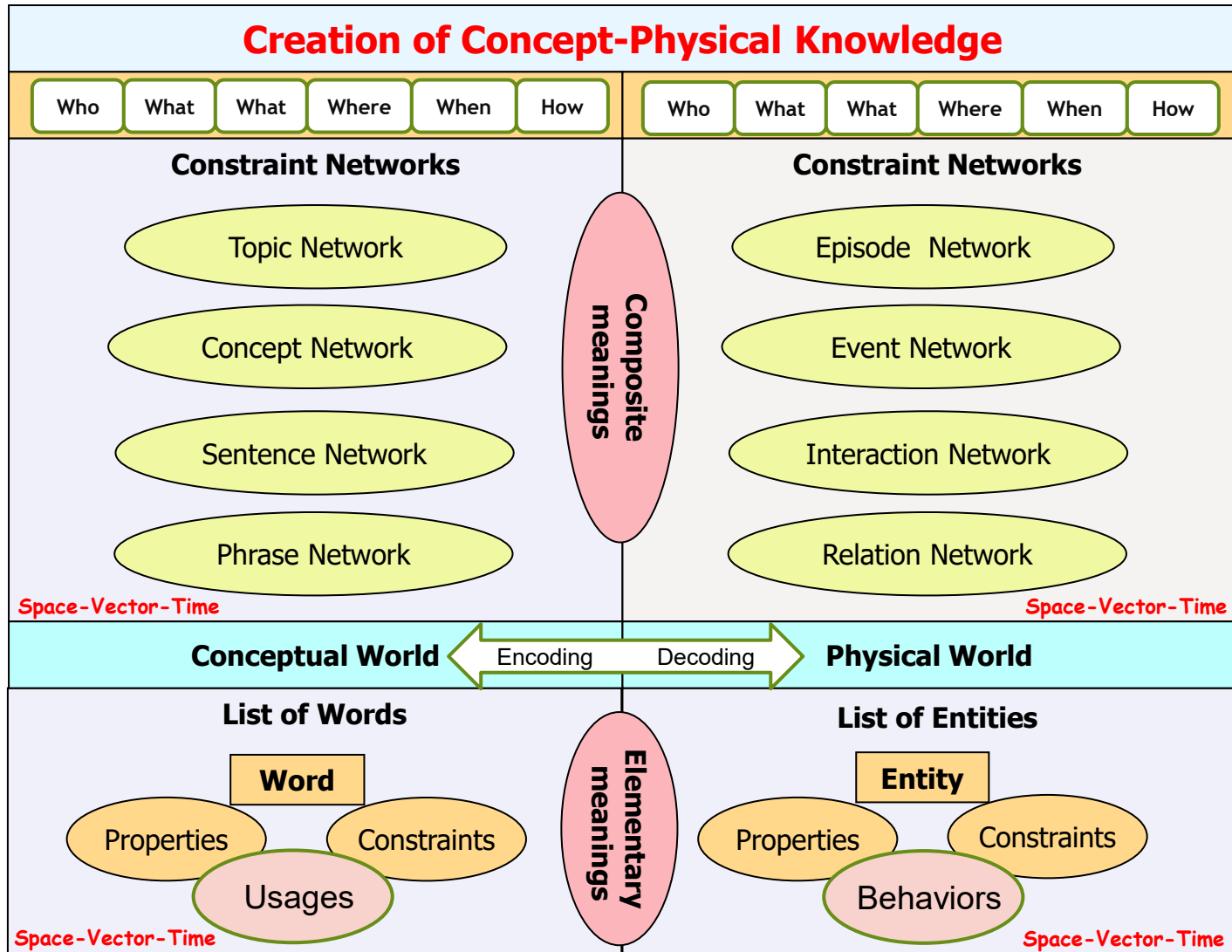
<http://personal.ntu.edu.sg/mmxie>

Warm-Up Questions ...

Are human being's sensory systems crisp or fuzzy?

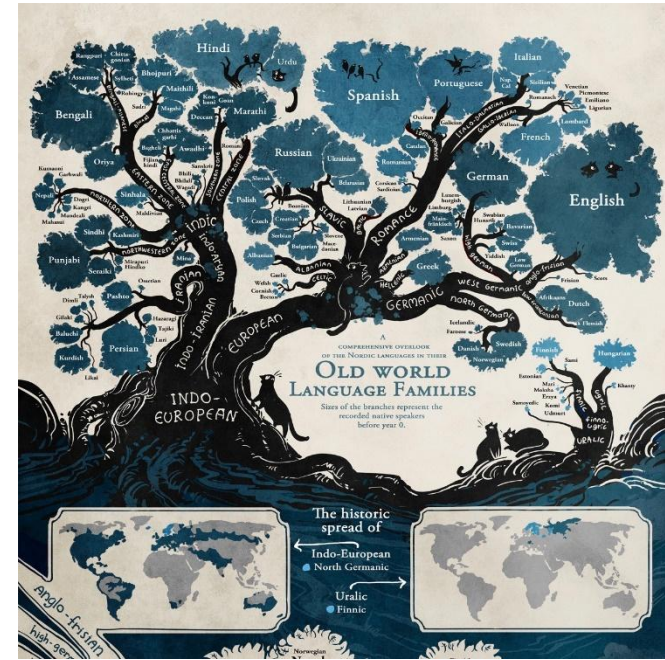


What are the magic outcomes from fuzzy sensory systems?



Outline

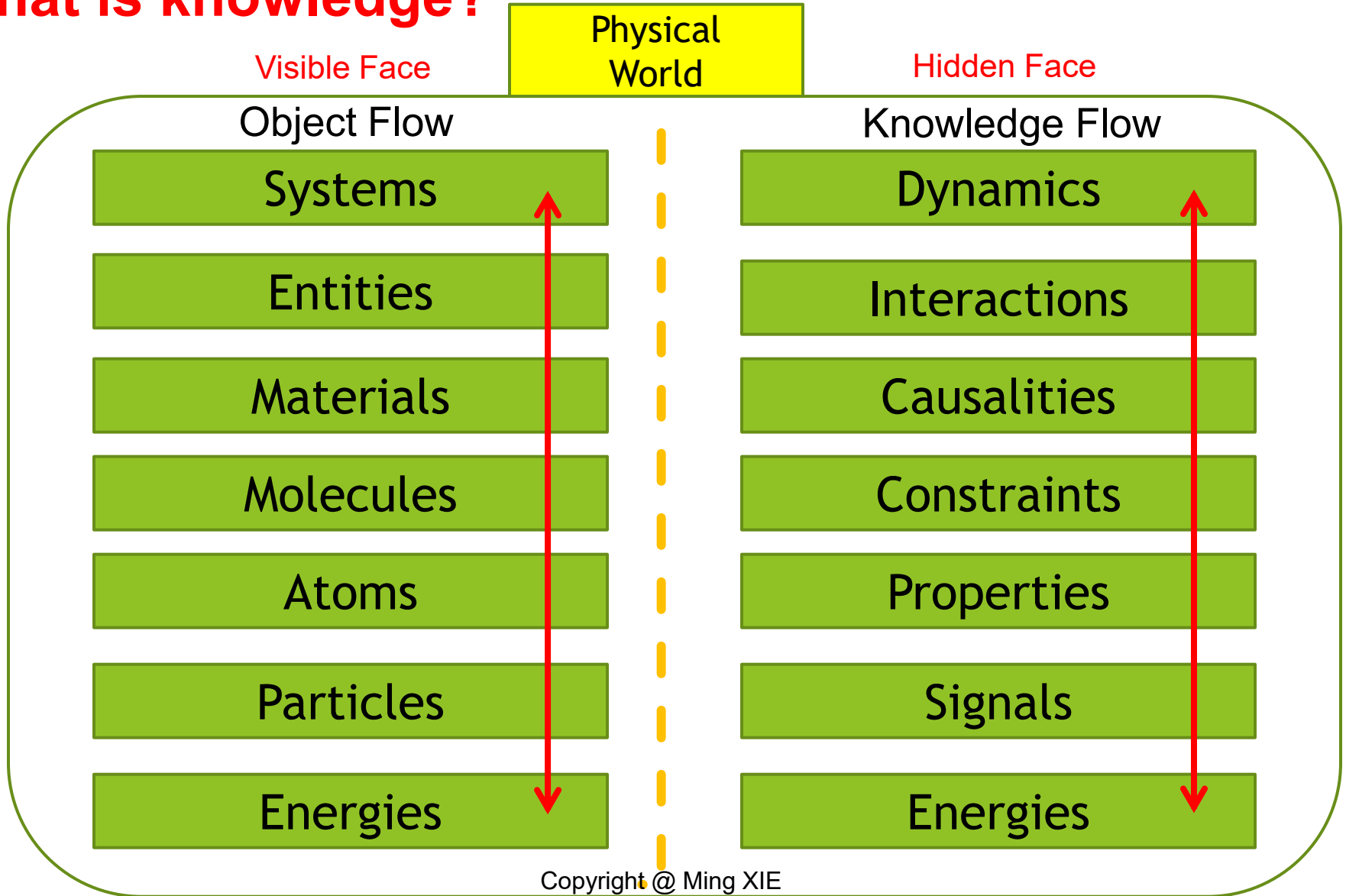
- ▶ Representation of Knowledge
 - ▶ Definition of Conceptual Worlds
 - ▶ Creation of Conceptual Worlds
-
- ▶ Invention of Natural Languages
 - ▶ Invention of Technical Language
 - ▶ Invention of Programming Languages



Invention of Human Languages

(a language is a virtual sensor which produces outputs of symbols)

What is knowledge?



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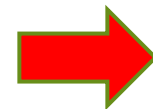
What is the universe?

▶ The universe consists of three types of world:

▶ The world of matters (flow of objects)

Its digital twin is the set of virtual scenes

▶ The world of laws (flow of knowledges)



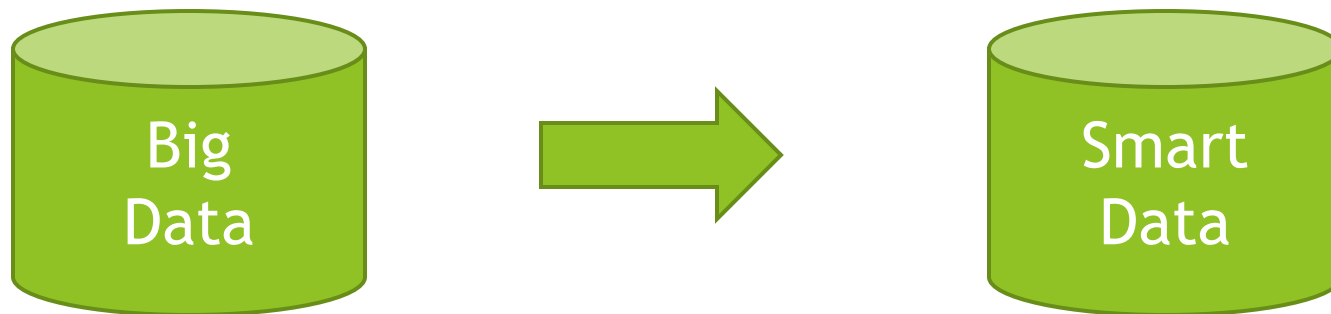
Next Slide

Its digital twin is the set of conceptual worlds

▶ The world of lives (flow of souls)

What is the future challenge faced by measurement and sensing systems?

- ▶ The future challenge is to transform big data into smart data with embedded description of knowledge from **the universe**.



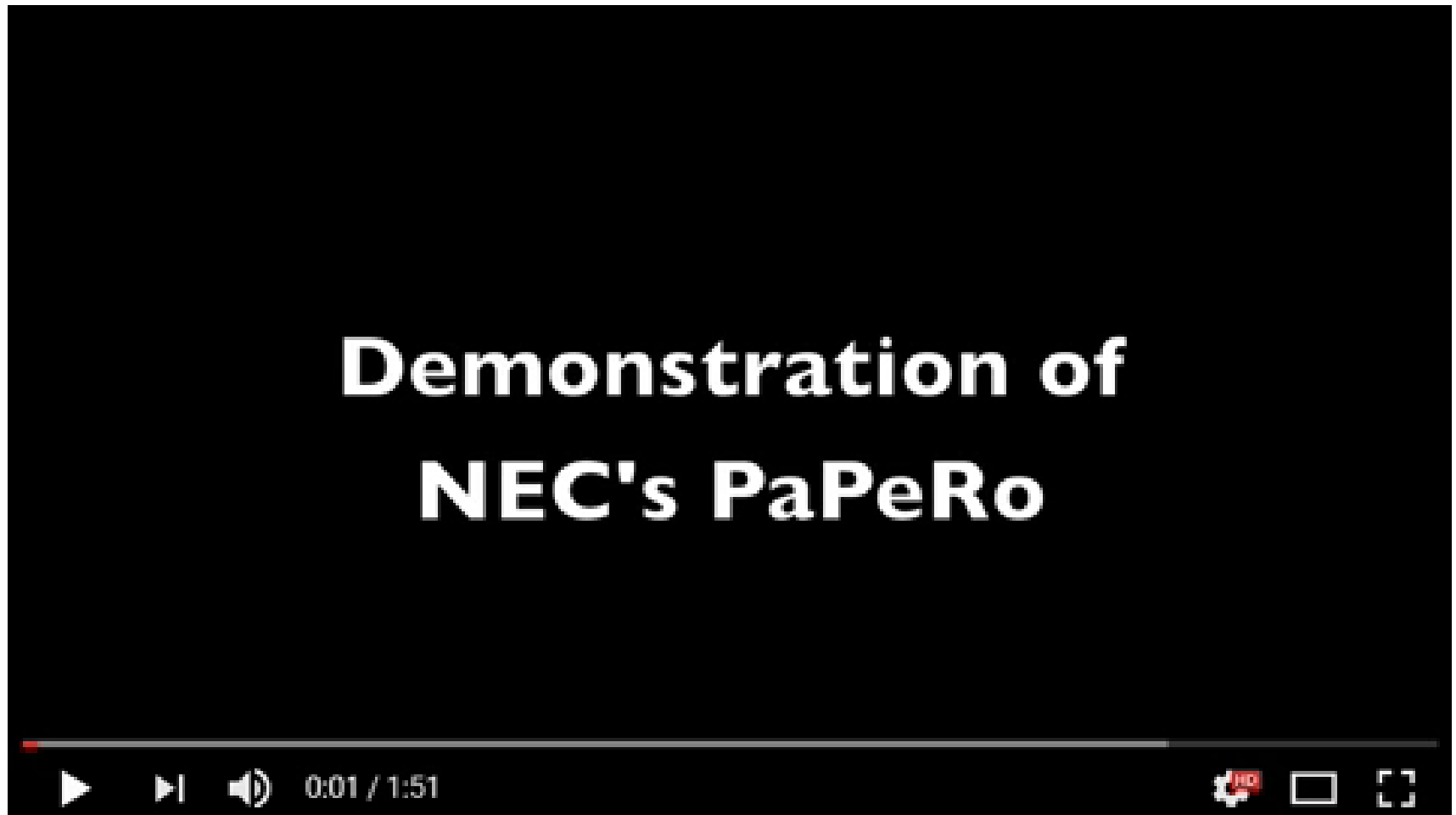
What is the best way to represent knowledge from the universe?

- ▶ The use of human languages which consist of:
 - ▶ Natural Languages
 - ▶ Technical Language
 - ▶ Programming Languages



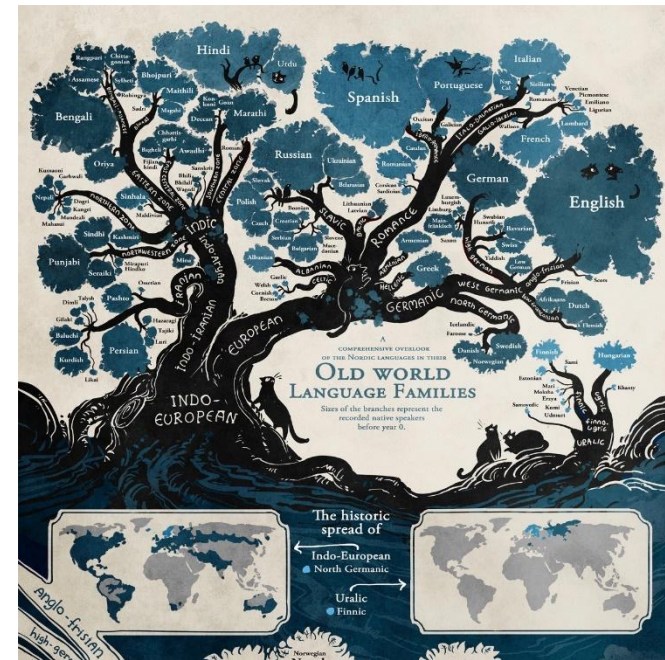
The use of human languages to represent knowledge creates conceptual worlds!

Example of Human-Robot Dialogue ...



Outline

- ▶ Representation of Knowledge
 - ▶ Definition of Conceptual Worlds
 - ▶ Creation of Conceptual Worlds
-
- ▶ Invention of Natural Languages
 - ▶ Invention of Technical Language
 - ▶ Invention of Programming Languages



Invention of Human Languages

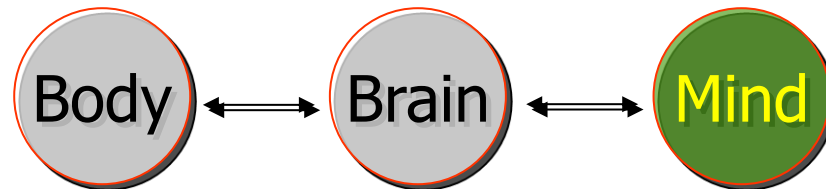
(a language is a virtual sensor which produces outputs of symbols)

Discussion 1

▶ A human being can learn any human language.

▶ Is it true?

▶ Why?



- Action
- Reaction

- Memory
- Computation

- Cognition
- Recognition

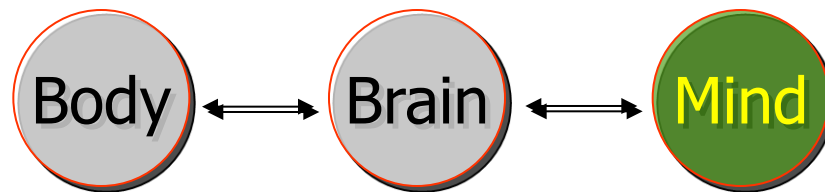
Discussion 2

- ▶ An animal, such as dog, cat and others, can never master a human language regardless of the effort of teaching.

- ▶ Is it true?



- ▶ Why?



- Action
- Reaction

- Memory
- Computation

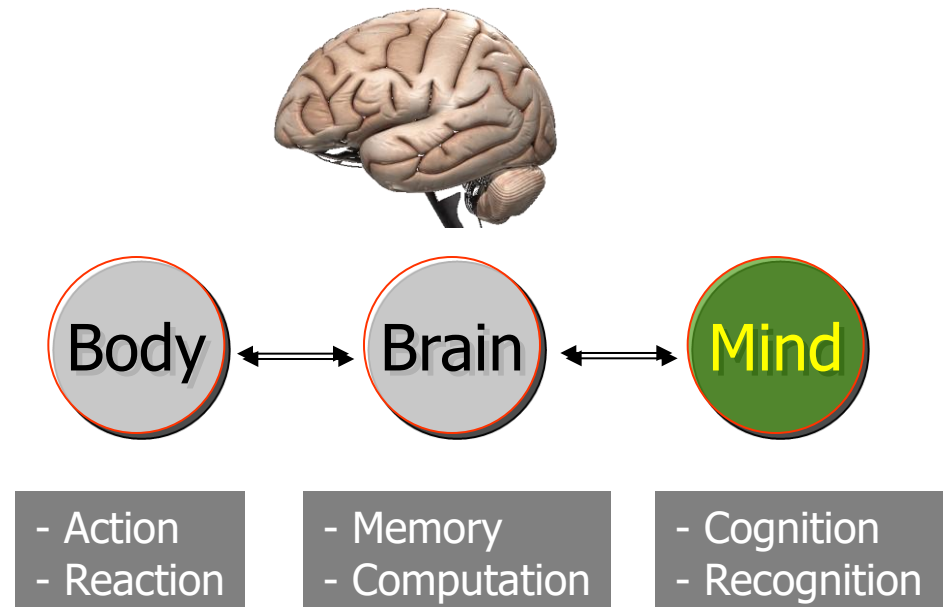
- Cognition
- Recognition

Discussion 3

► What is the greatest invention in human history?

► It is human language.

► Why?



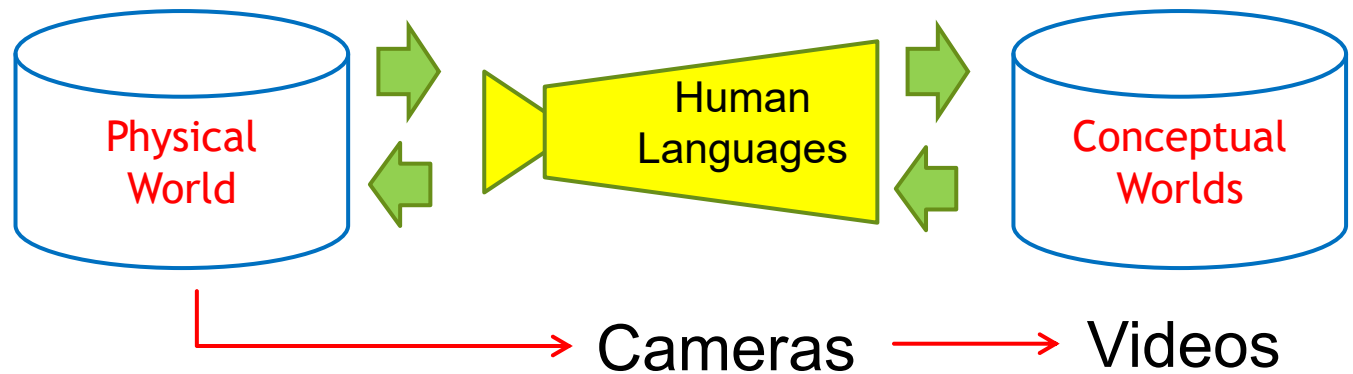
Discussion 4

- ▶ What is the common way of representing knowledge?
- ▶ It is the use of human languages, which produces texts.
- ▶ Why?



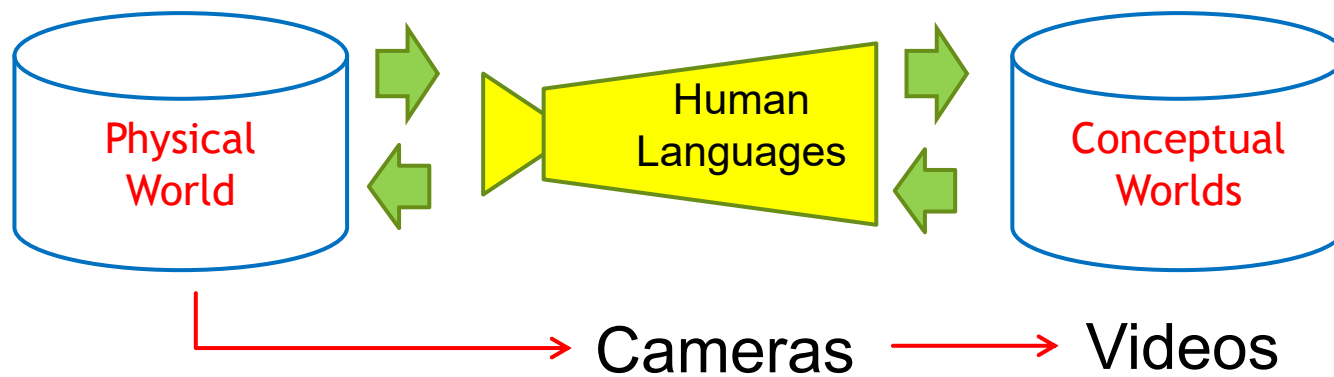
Discussion 5

- ▶ What is a human language?
- ▶ It is a set of words and rules for the purpose of projecting knowledge from the physical world into language-specific texts which form a conceptual world.
- ▶ Why?



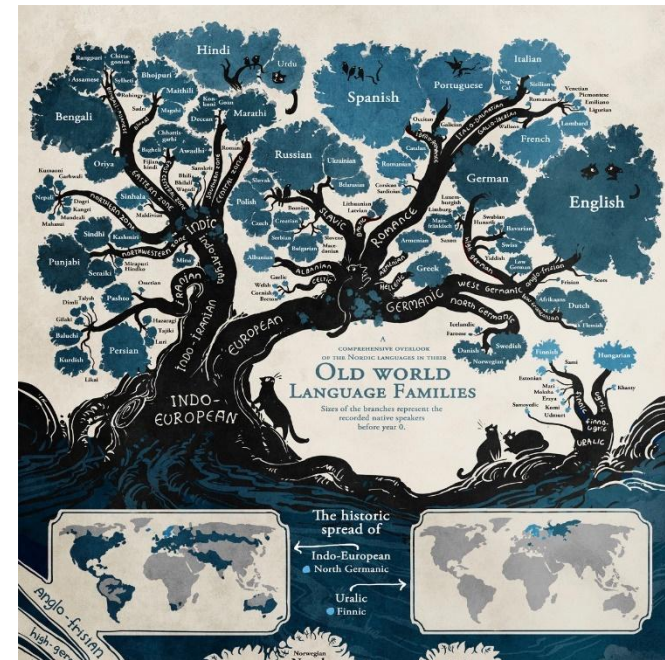
Definition of Conceptual Worlds

- ▶ The description of the physical world's knowledge in terms of properties, constraints and behaviors in the form of texts of a human language is called a conceptual world.



Outline

- ▶ Representation of Knowledge
 - ▶ Definition of Conceptual Worlds
 - ▶ Creation of Conceptual Worlds
-
- ▶ Invention of Natural Languages
 - ▶ Invention of Technical Language
 - ▶ Invention of Programming Languages



Invention of Human Languages

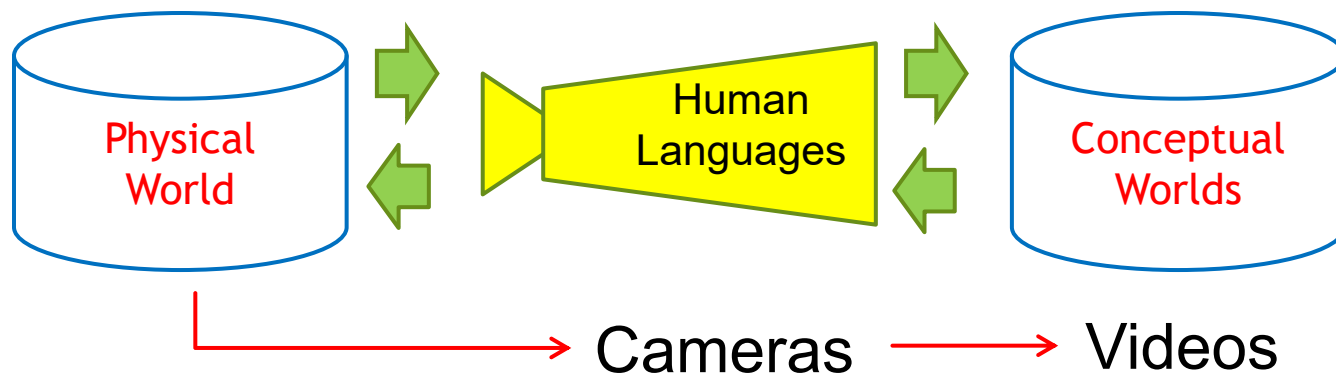
(a language is a virtual sensor which produces outputs of symbols)

There Are Two Major Applications with the Use of Human Languages

- ▶ Description of Knowledge
 - ▶ From meanings in physical world to meanings in conceptual worlds (symbol grounding)
- ▶ Understanding of Knowledge
 - ▶ From meanings in conceptual worlds to meanings in physical world (mental imagination)

What is knowledge description?

- It is the process of describing the knowledge of the physical world with the texts in human languages.



Example of Using Natural Language

- ▶ Description of a dynamic scene:

Physical World



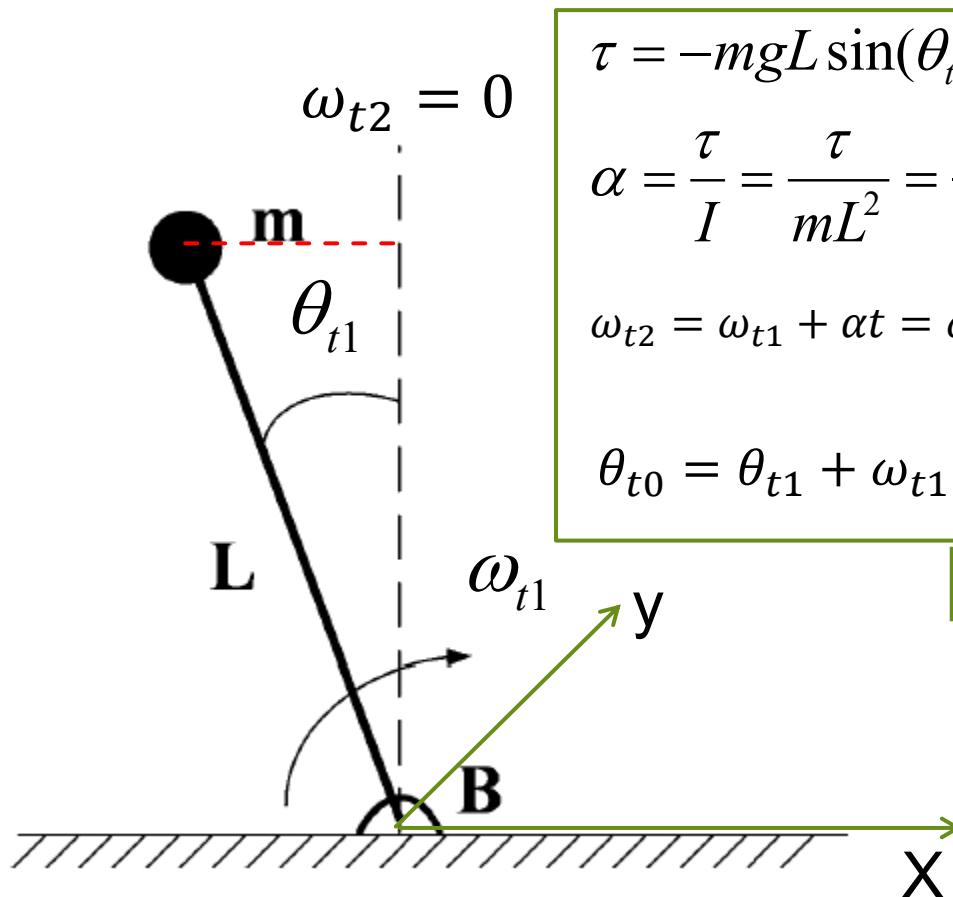
Conceptual World

A car is moving
on a mountain
road.



Example of Using Technical Language

- How to determine an **acceptable** initial angular velocity at which the mass will swing over from the left to the right?



$\tau = -mgL \sin(\theta_{t1})$ Approximate solution

$$\alpha = \frac{\tau}{I} = \frac{\tau}{mL^2} = -\frac{g \sin(\theta_{t1})}{L}$$

$$\omega_{t2} = \omega_{t1} + \alpha t = \omega_{t1} - \left[\frac{g}{L} \sin(\theta_{t1})\right] \cdot t$$

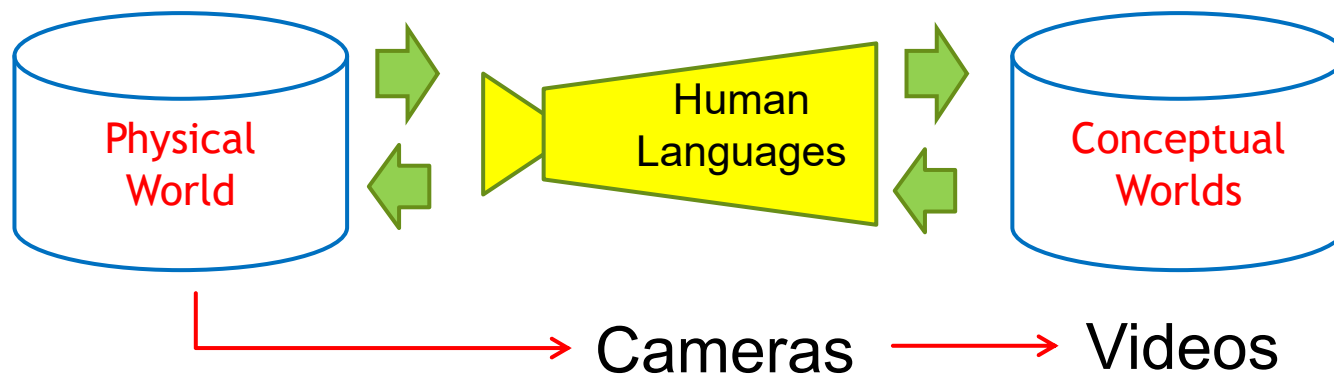
$$\theta_{t0} = \theta_{t1} + \omega_{t1}t + \frac{1}{2}\alpha t^2$$

$$\omega_{t1} - \left[\frac{g}{L} \sin(\theta_{t1})\right] \cdot t = 0$$

$$\theta_{t1} + \omega_{t1}t + \frac{1}{2}\alpha t^2 = 0$$

What is knowledge understanding?

- ▶ It is the process of reconstructing the knowledge of the physical world from texts in human languages.

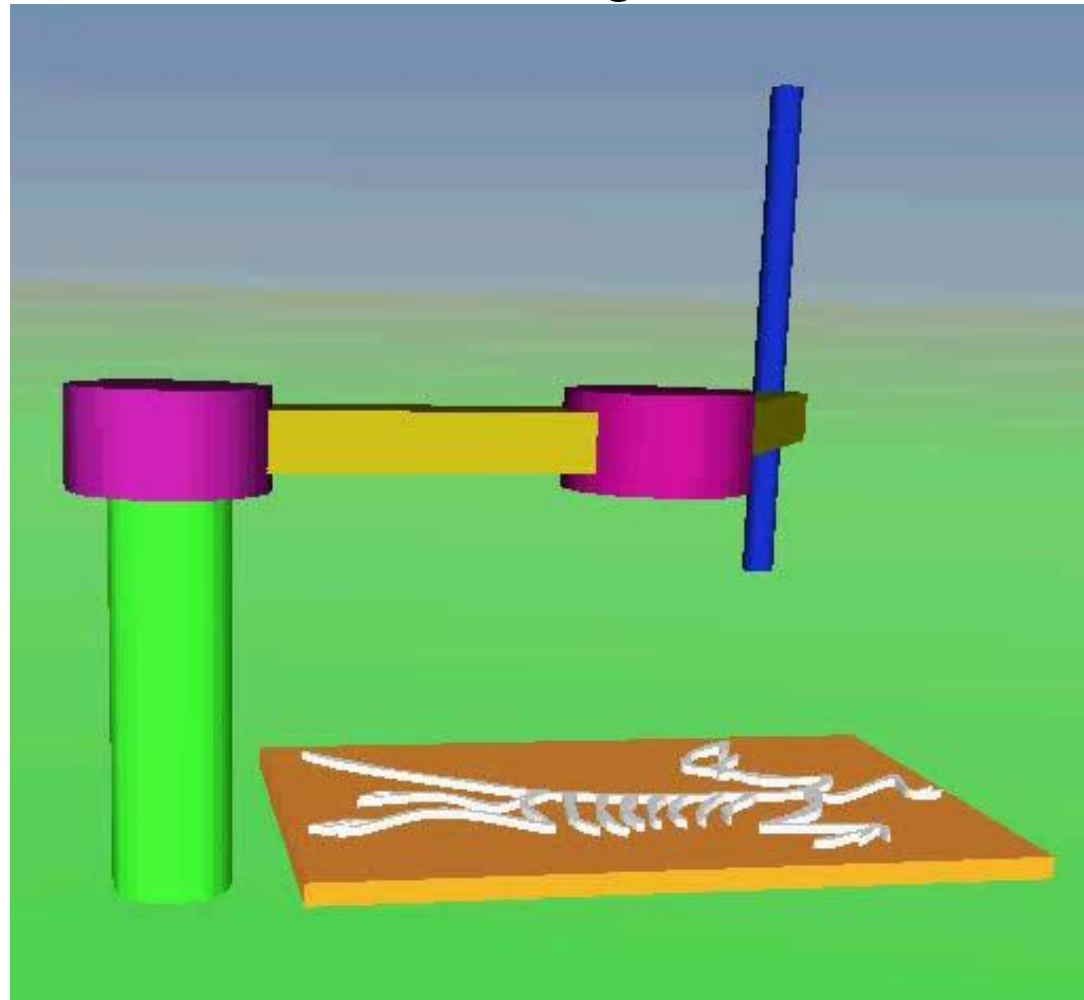


Example of Understanding Natural Language

Visualized Meanings inside Mind

A robot arm with two links, holds a laser cutting gun. There is a metal sheet which is placed within the workspace of the robot arm. A program is loaded into the robot and guides the robot to cut a shape of dinosaur on the metal sheet. Now, the robot is powered on and it starts the cutting process ...

Conceptual World



Hence, the Use of Human Languages Actually Creates Conceptual Worlds ...

- ▶ The texts in any human language are considered as a conceptual world because they describe the clusters of properties, constraints, behaviors, events and episodes in the physical world.
- ▶ A text is composed of
 - ▶ Sentences, and
 - ▶ Questions.

Models of Sentences in English

- ▶ Noun phrase + Verb Phrase + Noun Phrase
 - ▶ He asks Teacher for helps.
- ▶ Subject + Verb + Object 1 / Object 2 ...
- ▶ Subject + Verb + Indirect Object + Direct Object
 - ▶ The teacher gives us three assignments.
 - ▶ The robot brings me a drink.
- ▶ Subject + Verb + Object + Object Complement
 - ▶ We complete the assignments timely.

Subject + Verb + Object + Adverbial (Adverb/Preposition)

He finished all the lectures happily yesterday.
The robot picks up a key from a table.

Types of Sentences in English ...

- ▶ **Simple sentences**
 - ▶ The robot starts working at 8h30 daily.
- ▶ **Compound sentences**
 - ▶ Robot A is doing assembly and robot B is doing painting.
- ▶ **Complex sentences**
 - ▶ I look at the robot which is doing material handling.
- ▶ **Requests**
 - ▶ Enjoy your study! Have a nice day! Hand in the reports by 2 pm.
- ▶ **Commands**
 - ▶ Finish the works by 5 pm! Enter the building! Switch off the light!
- ▶ **Exclamations**
 - ▶ What a wonderful robot! Isn't the robot crazy!

Models of Questions in English

▶ Yes/No Questions

▶ Verb + Subject

- ▶ Is this machine a robot?
- ▶ Can a robot kill us?

▶ WH- Questions

▶ Verb + Subject + Verb

- ▶ Who give you that robot?
- ▶ How many hours can the robot work?

▶ Tag Questions

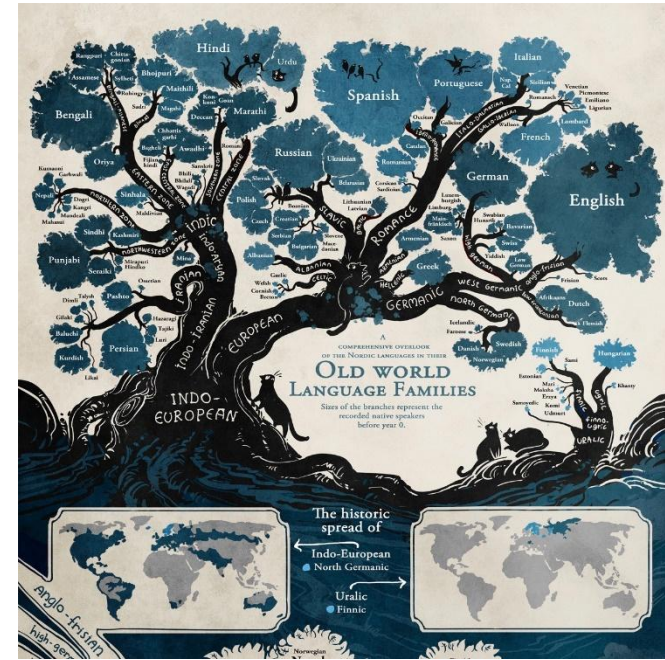
▶ Declarative sentence + Question Tag

- ▶ A robot can do dirty jobs, isn't it?
- ▶ He has not yet bought any book, has he?



Outline

- ▶ Representation of Knowledge
 - ▶ Definition of Conceptual Worlds
 - ▶ Creation of Conceptual Worlds
-
- ▶ Invention of Natural Languages
 - ▶ Invention of Technical Language
 - ▶ Invention of Programming Languages



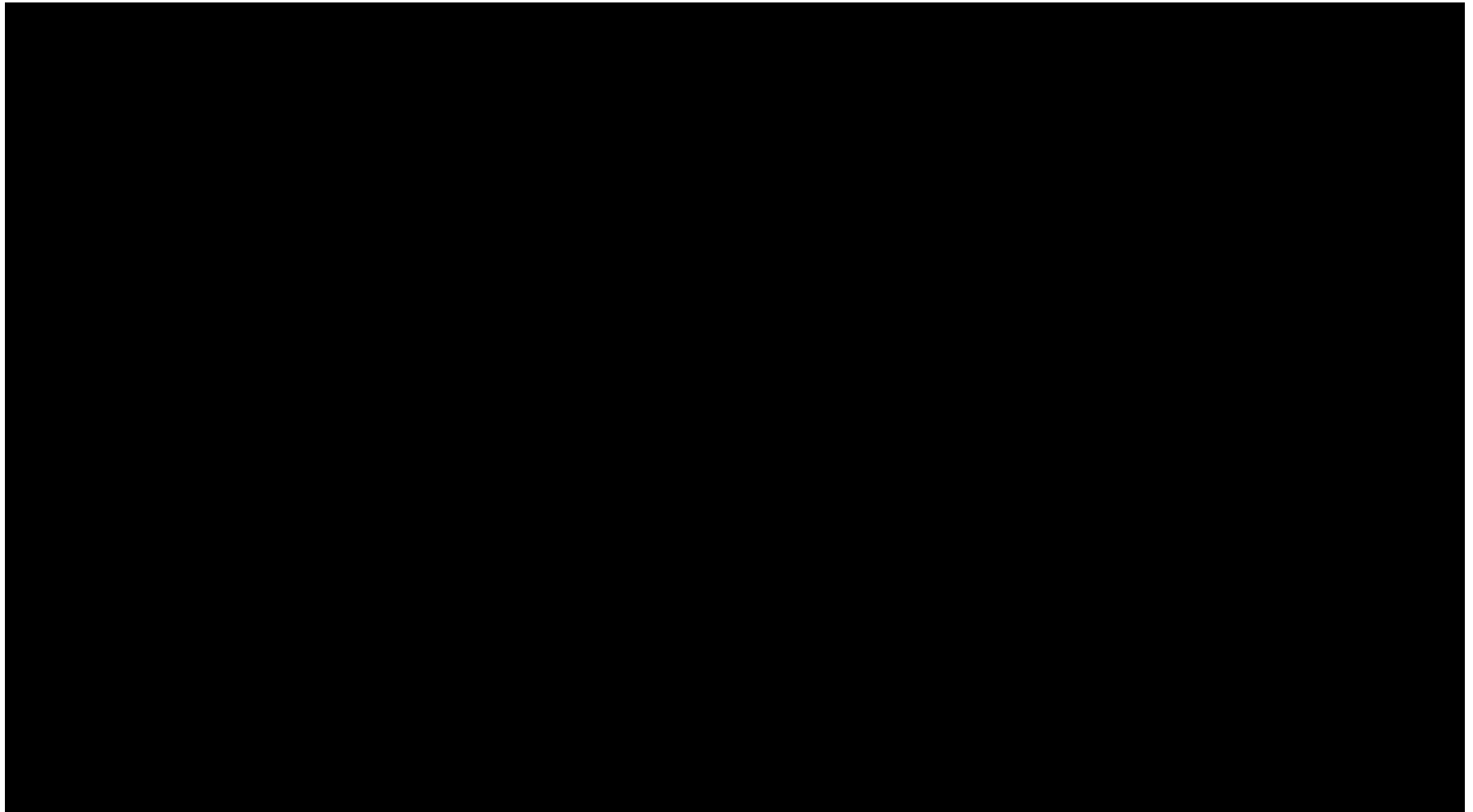
Invention of Human Languages

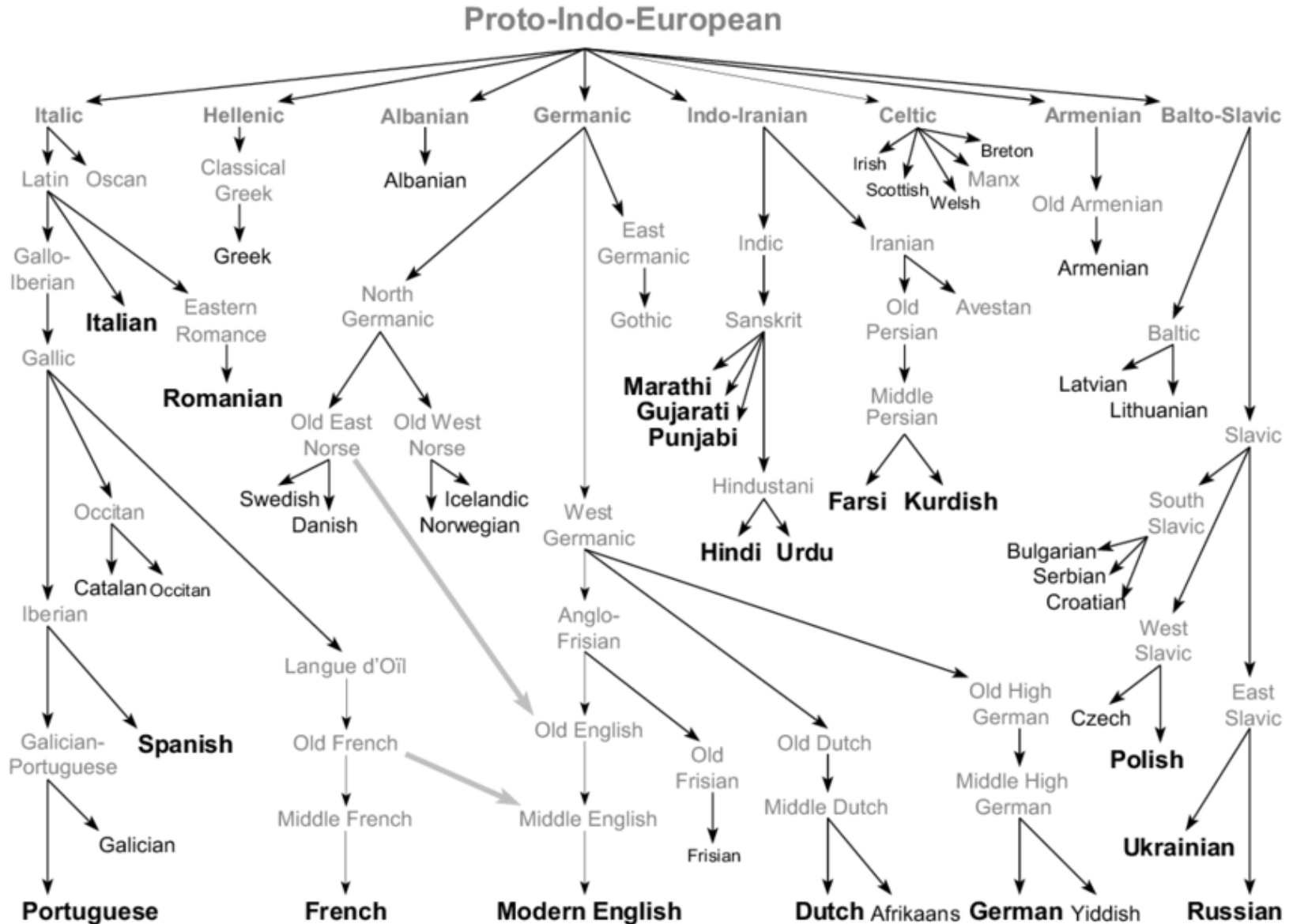
(a language is a virtual sensor which produces outputs of symbols)

A Statement ...

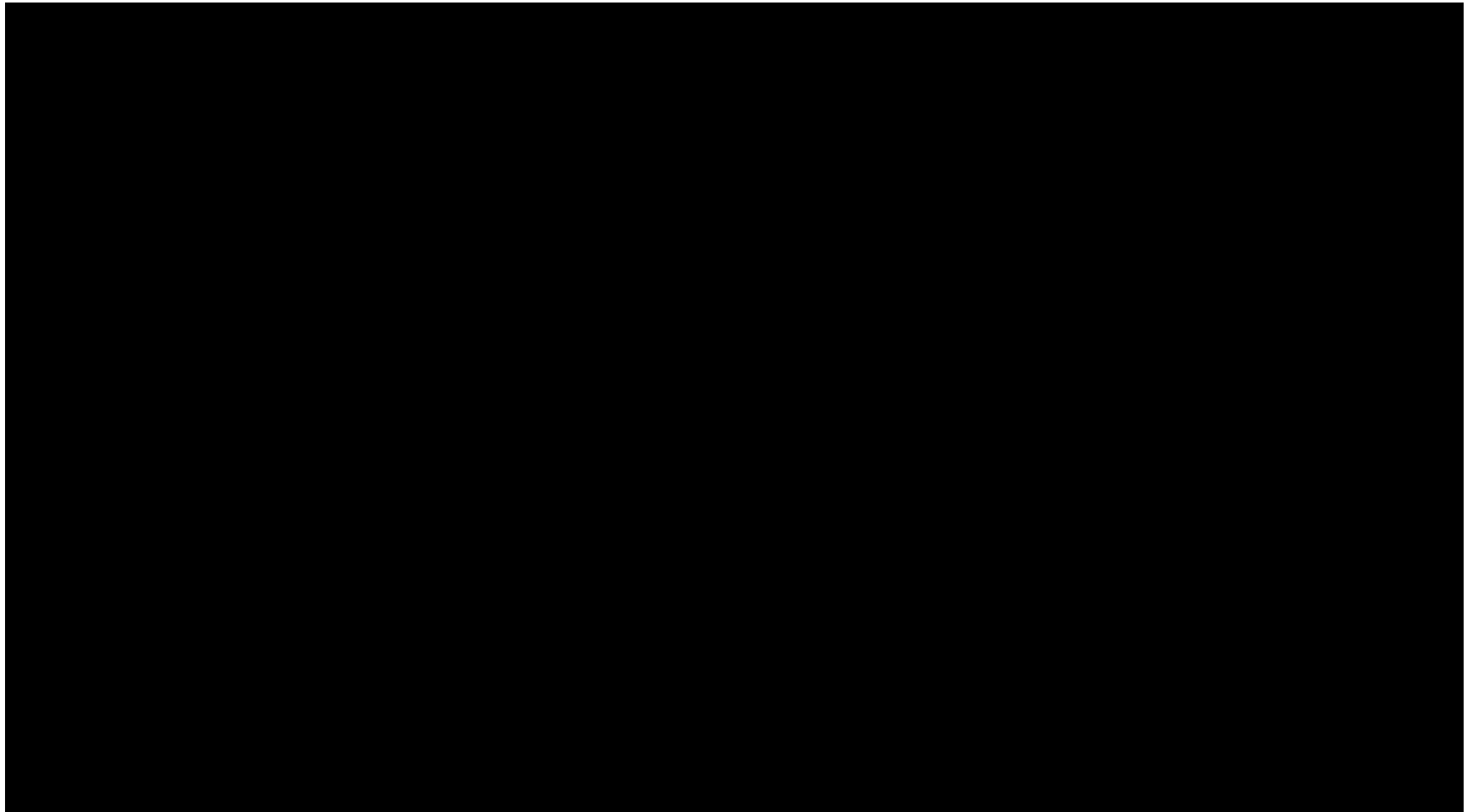
- ▶ **A human language**, which consists of natural language, technical language and programming languages, is the outcome of **invention** motivated by **discovery**. - Ming XIE

Evolution of Natural Languages ...



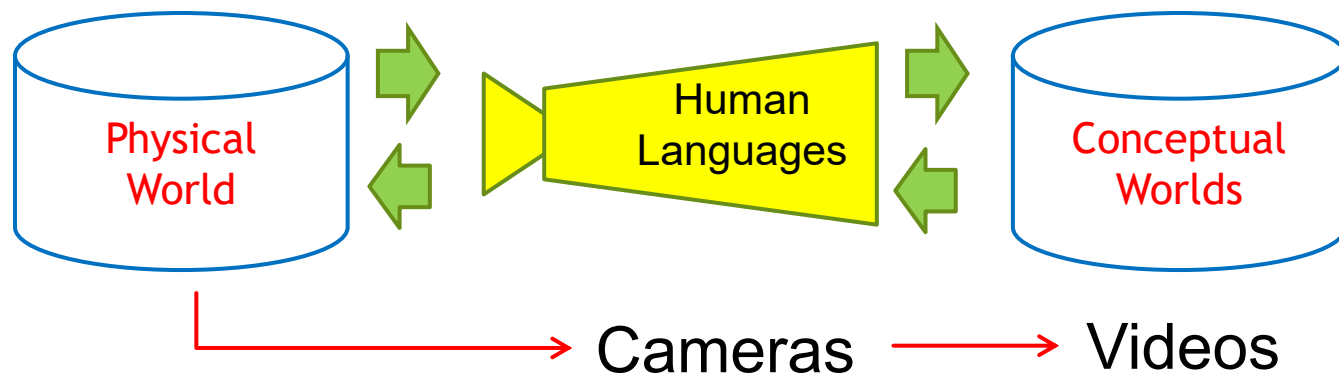


History of English Language ...



Definition of Natural Languages

- ▶ It is a set of words and rules for the purpose of **projecting** knowledge of the physical world into texts which form conceptual worlds.



Example of Using Natural Language ...

April 2024



1990s



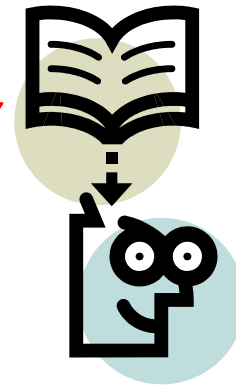
Japan

You see it in Japan.

How to tell your friends in Singapore?



Singapore

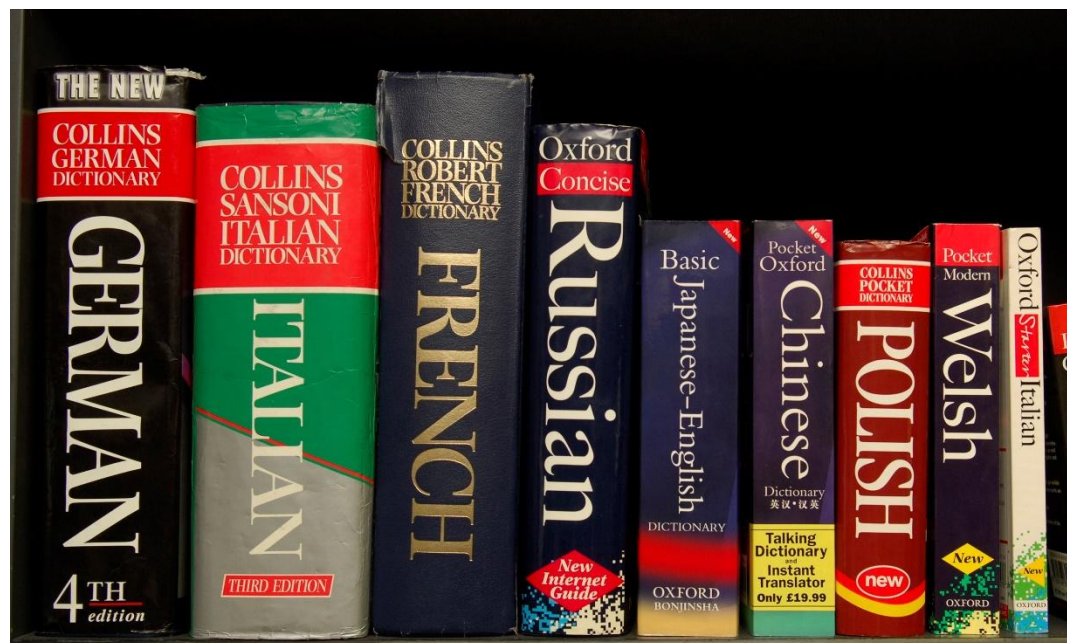


Definition of Word

- ▶ A word is a string of characters (or strokes or others), which refers to a specific meaning in the physical world.
- ▶ Each word has:
 1. A visible string of characters
 2. Invisible types or constraints

Definition of Vocabulary

- ▶ A set of words in a natural language is called the vocabulary.



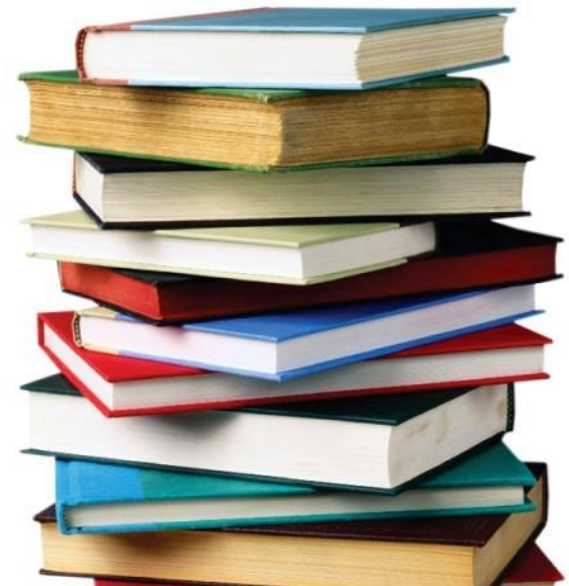
Definition of Grammar

- ▶ The set of rules, which govern the properties and constraints of words in a natural language, is called grammar.

- ▶ Grammar imposes:
 1. Rules of constraining words
 2. Rules of combining words

What are the primary needs behind the invention of natural languages?

- ▶ To represent and record knowledge
- ▶ To communicate knowledge across space:
 1. Long distance
 2. Short distance
- ▶ To communicate knowledge across time:
 1. Instant conversation
 2. Deferred teaching and learning
- ▶ To support knowledge discovery and creation



How are natural languages been invented?

- ▶ The invention of a natural language is guided by the knowledge of the physical world.
- ▶ The knowledge of an entity includes:
 - ▶ Properties
 - ▶ Constraints
 - ▶ Behaviors

What is the reason behind the invention of “Noun”?

► It is the need of creating words which refer to

- | | | | |
|----|---------------------|---|---------------|
| 1. | physical properties | } | Concrete noun |
| 2. | physical behaviors | | |
| 3. | physical events | } | Abstract noun |
| 4. | physical processes | | |
| 5. | physical episodes | | |

Example

Grape

Pineapple

Fruits



Chili



Water melon



Apple

Mango

Types of Noun in English

- ▶ Common nouns (such as: Jack, John, ...)
- ▶ Collective Nouns (such as : team, committee, army, ...)
- ▶ Compound Nouns (such as: sister-in-law, hard-disk, ...)
- ▶ Singular Nouns (such as: Singapore, China, Japan, France, ...)
- ▶ Plural Nouns (such as: cars, cows, rooms, buildings, ...)

What is the reason behind the invention of “Noun Phrase”?

- ▶ It is the need of using a set of words which refer to relationship among
 1. Physical properties,
 2. Physical behaviors,
 3. Events,
 4. Processes, and
 5. Episodes, etc

How to form Noun Phrase with Pre-modifiers?

▶ To use determiner (word type)

▶ Demonstrative determiners



That car

▶ Possessive determiners



Our car

▶ Interrogative determiners



Whose car

▶ To use adjective (word type)



Red cars



- Metal bar

- School uniform

▶ To use additional noun



- A lot of robots

▶ To use quantifiers (word type)



Three cars

- A little time

- Many books

How to form Noun Phrase with Post-modifiers?

- ▶ To use prepositions (word type)
 - The teacher in red hat
 - The book about robotics
 - The building with green windows

- ▶ To use relative clauses
 - The student whose brother is soldier
 - The worker whose wife is doctor

- ▶ To use non-finite clauses
 - The researcher to give a talk
 - The driver wearing a dark glasses
 - The robot kicking the ball

What is the reason behind the invention of “Adjective”?

- ▶ It is the need of creating words which refer to the constraints (both physical constraints and mental constraints (opinions)) about:
 - ▶ Size (tiny, small, big, huge, ...)
 - ▶ Shape (triangular, rectangular, circular, ...)
 - ▶ Age (young, old, ...)
 - ▶ Color (red, green, blue, ...)
 - ▶ Origin (near, far, close, ...)
 - ▶ Material (solid, liquid, gas, ...)
 - ▶ Behavior (slow, fast, violent, gentle, ...)
 - ▶ Event (terrible, wonderful, ...)
 - ▶ Process (random, stationary, ...)
 - ▶ Episode (exciting, interesting, ...), and others

What is the reason behind the invention of “Pronoun”?

- ▶ The behavior of an entity can evolve in time and in space.
- ▶ The interaction among entities can also evolve in time and in space.
- ▶ Hence, there is a need of tracking the reference to:
 1. Properties/behaviors of a single entity
 2. Properties/constraints/behaviors/interactions/events/episodes/relationship among entities.

Types of Pronouns

▶ Personal pronouns	→	I, you, he, she, me, us, them
▶ Possessive pronouns	→	mine, yours, ours, hers, his
▶ Interrogative pronouns	→	which, whose, what, how much
▶ Relative pronouns	→	which/that, whose, who, whom
▶ Demonstrative pronouns	→	this, that, these, those
▶ Reflexive pronouns	→	myself, yourself, themselves
▶ Indefinite pronouns	→	all, any, anybody, nothing, none, each
▶ Reciprocal pronouns	→	each other, one another

What is the reason behind the invention of “Verb”?

- ▶ All behaviors/interactions/events/episodes occur in time and in space.
- ▶ The need of describing time-related behaviors/interactions/events/episodes motivates people to invent words of the type “Verb”.

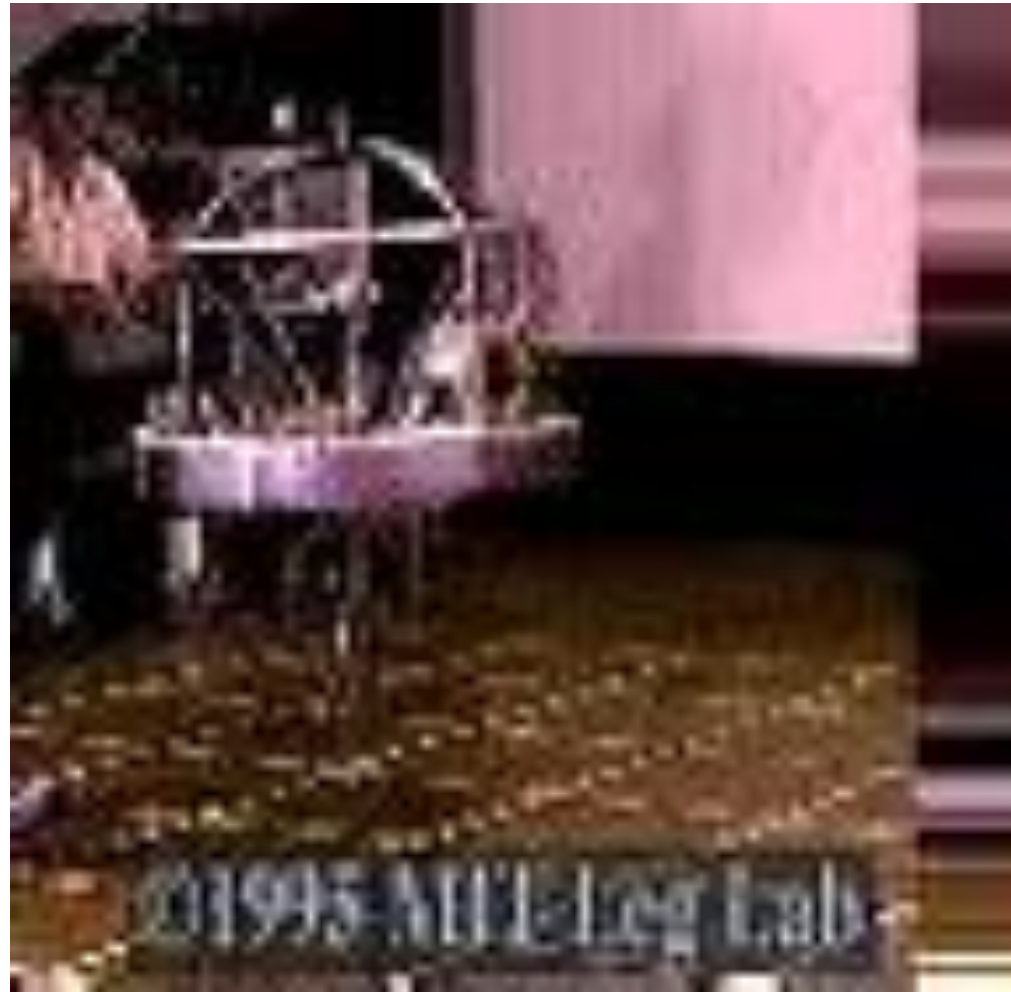
Example: Launch

Canister Launch from a Moving
Armoured Vehicle

Example: Flying, has flied, will fly



Example: Jump



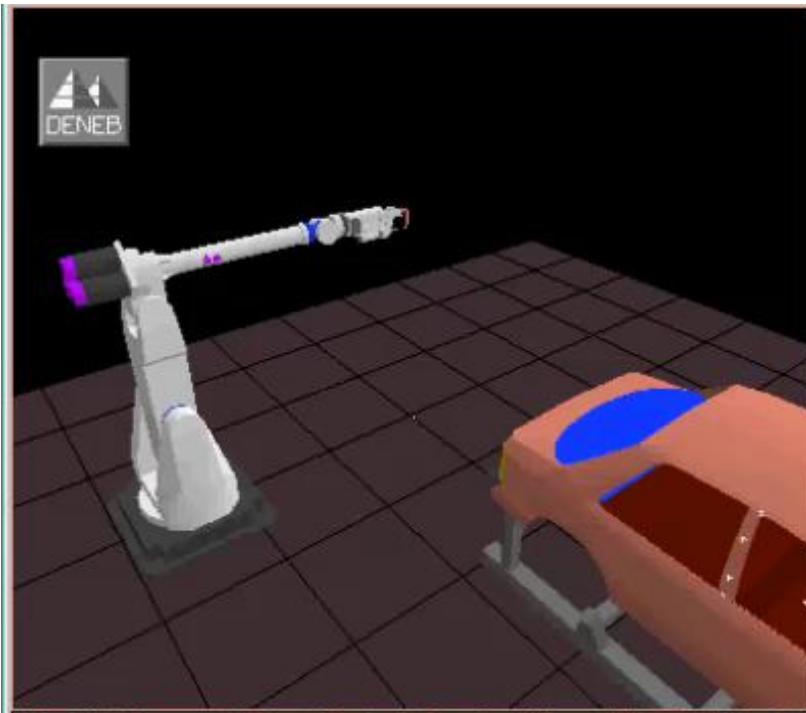
Types of Verbs

- ▶ Action Verbs: Jump, Fly, Hit, Push, Swing, Pull, Write, Close, Walk
- ▶ Saying Verbs: Say, Speak, Talk, Argue, Explain, Announce, Beg
- ▶ Sensing Verbs: Smell, Feel, Hear, Touch, Observe, Perceive, Read
- ▶ Mental Verbs: Think, Infer, Reason, Solve, Calculate, Decide
- ▶ Linking Verbs: Be, Seem, Become, Appear, Have
- ▶ Modal Verbs: Will, Must, Might, Would, May, Can, ...
- ▶ Special Linking Verbs (notion of helping verbs):
 - ▶ Be : These are my sisters. We are from Singapore. It is 2pm.
 - ▶ Have : It has two legs. I have a car. I am having headache.

What is the reason behind the invention of “Verb Phrase”?

- ▶ When there are multiple entities in time and in space, there is a need to use words to describe:
 1. Behaviors of entities in time and in space
 2. Interactions among entities in time and in space
 3. Events caused by entities in time and space
 4. Episode caused by entities in time and space

Example of Using Verb-Phrases ...



The body of a car is carried by an AGV (automatic guided vehicle), which approaches a spot-welding station. At the spot-welding station, a robot is to undertake the spot-welding at the predefined locations inside the body of the car. Once completed, the AGV will carry the car's body, and go to the next station.

Types of Verb Phrases in English

► Using the present tense

1. He likes study. She is doing a project in the lab.
2. The lecture has finished. The robot has started walking.

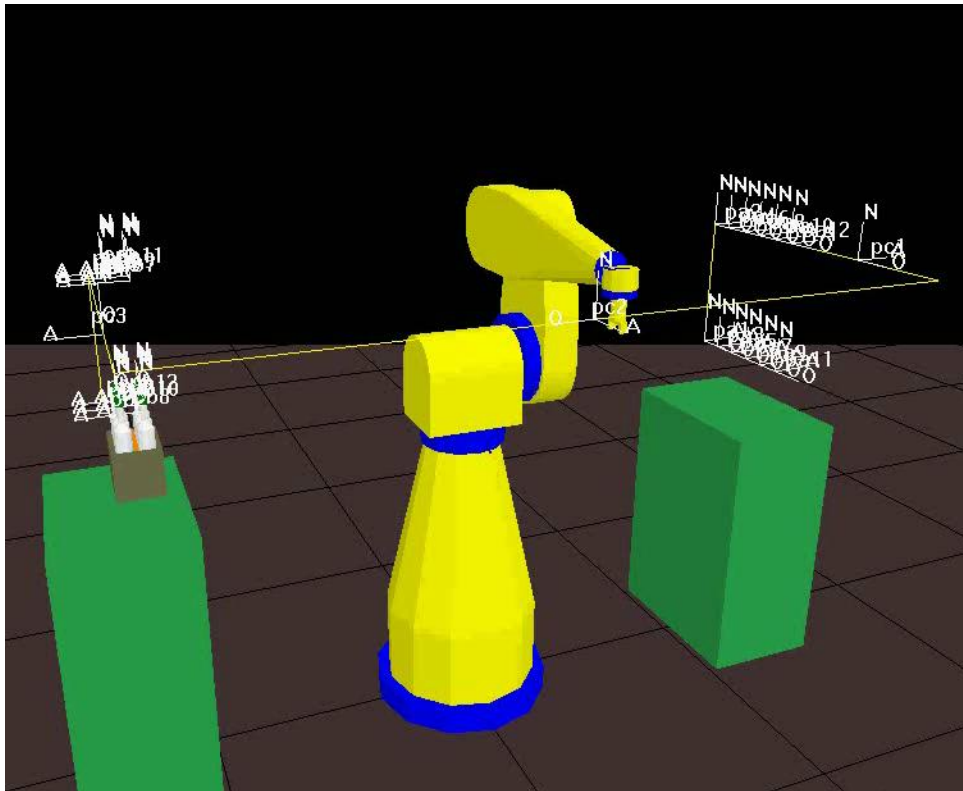
► Using the past tense

1. I sent you an email yesterday. I saw you on TV this morning.
2. The robot was working last night. He had been in Japan in 2000.

► Expressing future time

1. We will study lecture 11 next week. I shall travel to Europe soon.
2. She will be doing the project tomorrow.
3. He is going to submit the report next week.

Example of Using Verb-Phrases ...



Beverage bottles are filled and transported to a packaging station by a conveyor. At the packaging station, the beverage bottles are placed into boxes. Each box contains six beverage bottles. When a box is full, it will be shifted away by another conveyor, which will also bring in an empty box.

What is the reason behind the invention of “Adverb”?

- ▶ Actions of an entity occur in time and in space.
- ▶ Interactions among entities occur in time and in space.
- ▶ Actions and interactions may vary in intensity, willingness, and intention.
- ▶ Hence, there is a need of using words to describe the verbs' manner, degree, time, and place, etc.

Types of Adverbs in English

- ▶ Adverbs of manner
 - ▶ He easily understands your talk.
 - ▶ She writes beautifully.
 - ▶ The robot seats there quietly.
- ▶ Adverbs of degree
 - ▶ I completely agree with you.
 - ▶ The robot acts so powerfully.
 - ▶ He almost fails the exam. He is very happy.
- ▶ Adverbs of time
 - ▶ He will come tomorrow.
 - ▶ I saw him two days ago.
- ▶ Adverbs of place
 - ▶ A car moves down a hill.
 - ▶ A robot walks forward, turns left and then walks upstairs.

Adverbs of connection:

Firstly, I read.

Then, I write.

Finally, I present.

What is the reason behind the invention of “Preposition”?

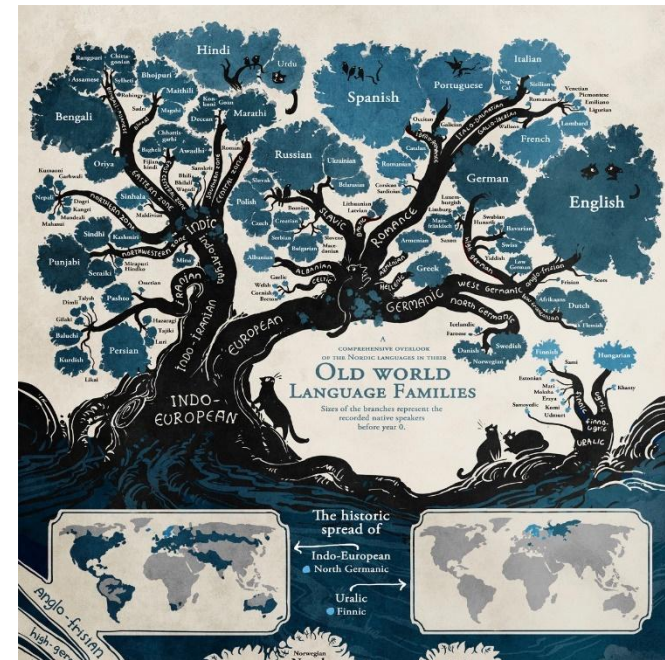
- ▶ Entities exist in time and space.
- ▶ Actions of an entity occur in time and in space.
- ▶ Interactions among entities occur in time and in space.
- ▶ Hence, there is a need of using words to describe the nouns’ and the verbs’:
 1. Time reference
 2. Place reference
 3. Direction reference
 4. General reference, etc

Types of prepositions in English

- ▶ Prepositions of **time reference**
 1. The lectures on Tuesday start at 4h30 pm.
 2. You have to submit your project report before 18 April 2013.
- ▶ Prepositions of **place reference**
 1. The robot puts its hands on top of a table.
 2. NTU is in Jurong West.
- ▶ Prepositions of **direction reference**
 1. The car moves along a road. The robot walks through a door.
 2. The bus heads toward NTU. I seats in his left side.
- ▶ Prepositions of **general reference**
 1. He gives a book to her. He plays outdoor regardless of weather.
 2. I will teach Part 1 about knowledge representation.

Outline

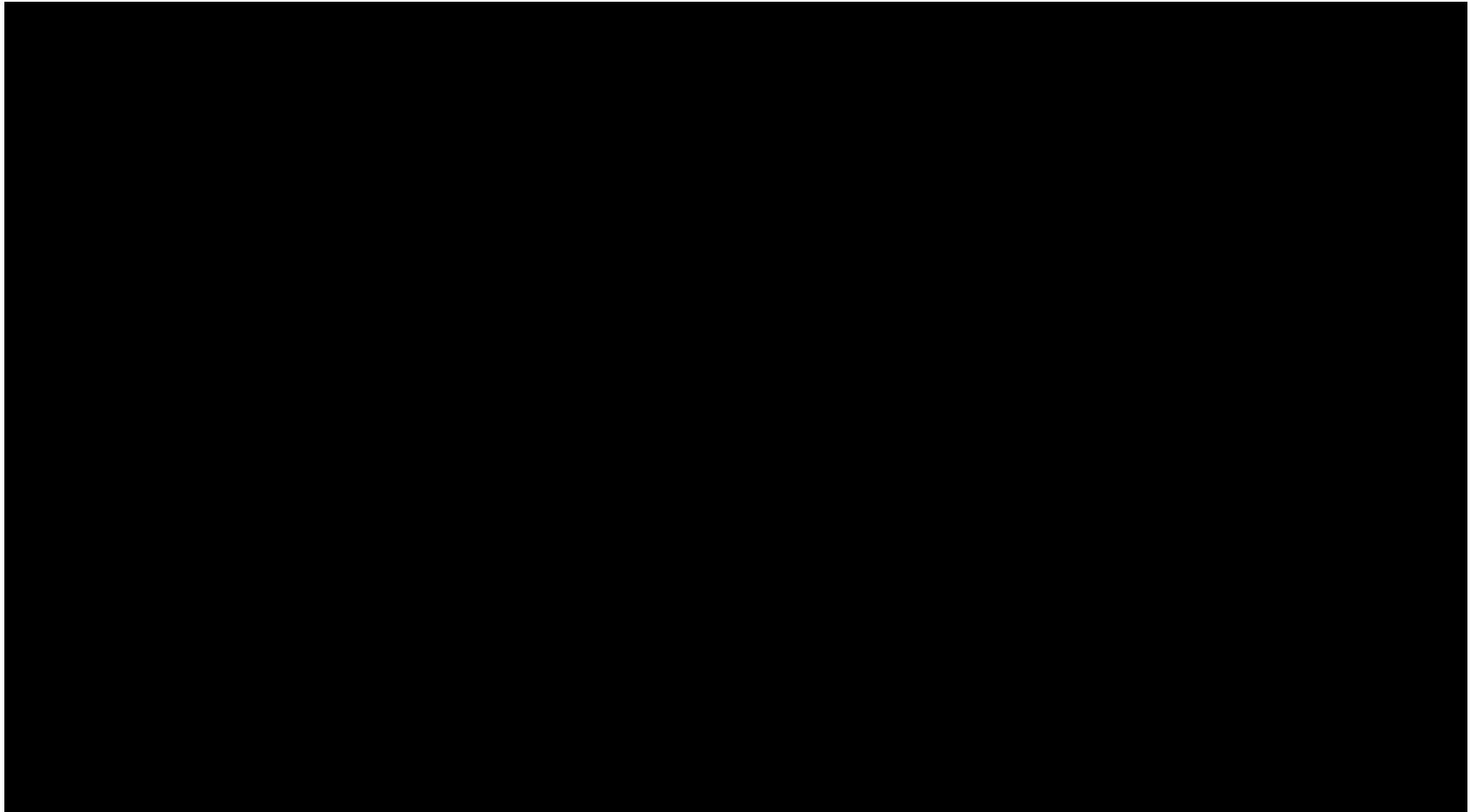
- ▶ Representation of Knowledge
 - ▶ Definition of Conceptual Worlds
 - ▶ Creation of Conceptual Worlds
-
- ▶ Invention of Natural Languages
 - ▶ Invention of Technical Language
 - ▶ Invention of Programming Languages



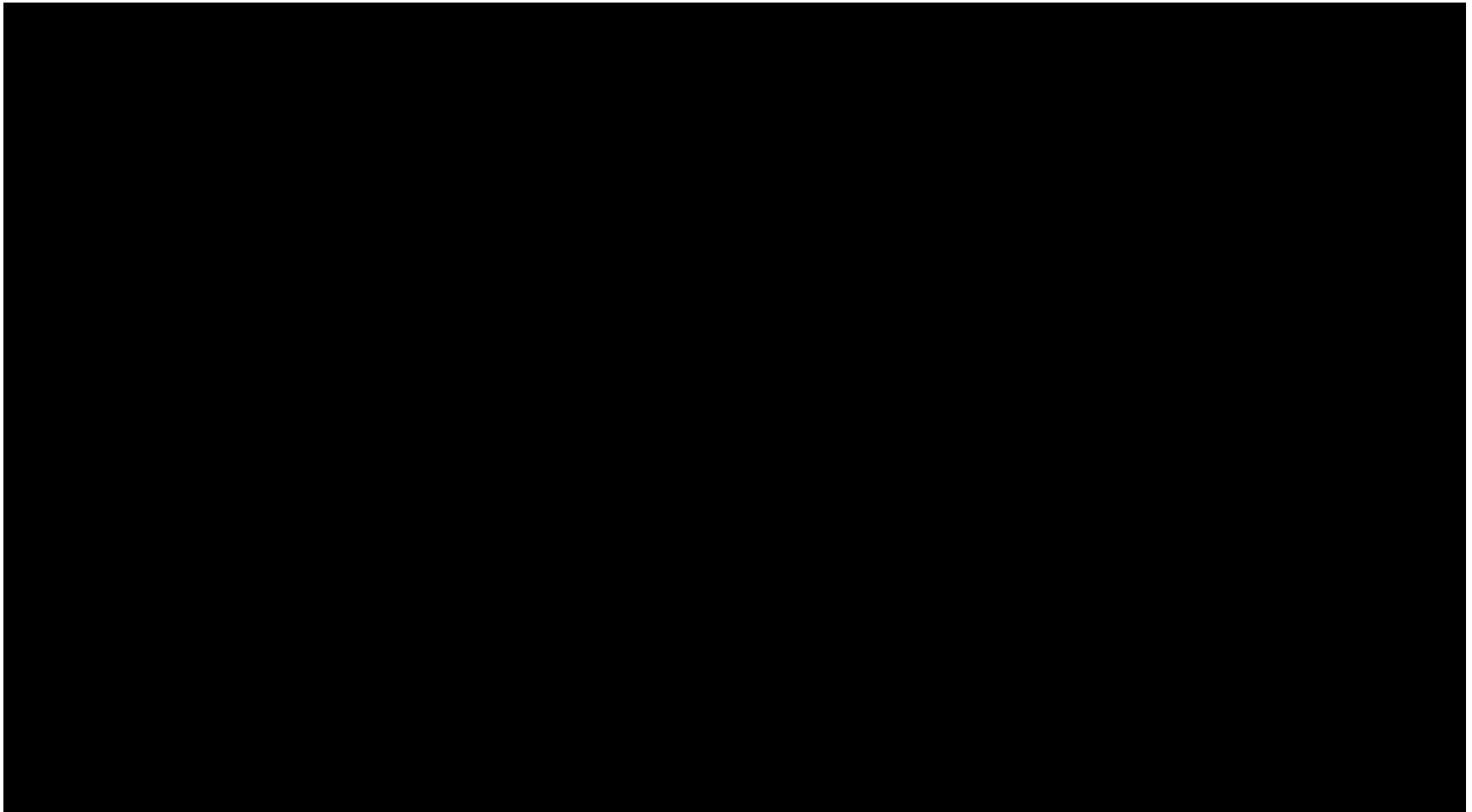
Invention of Human Languages

(a language is a virtual sensor which produces outputs of symbols)

Are technical languages invented?



How are technical languages invented?



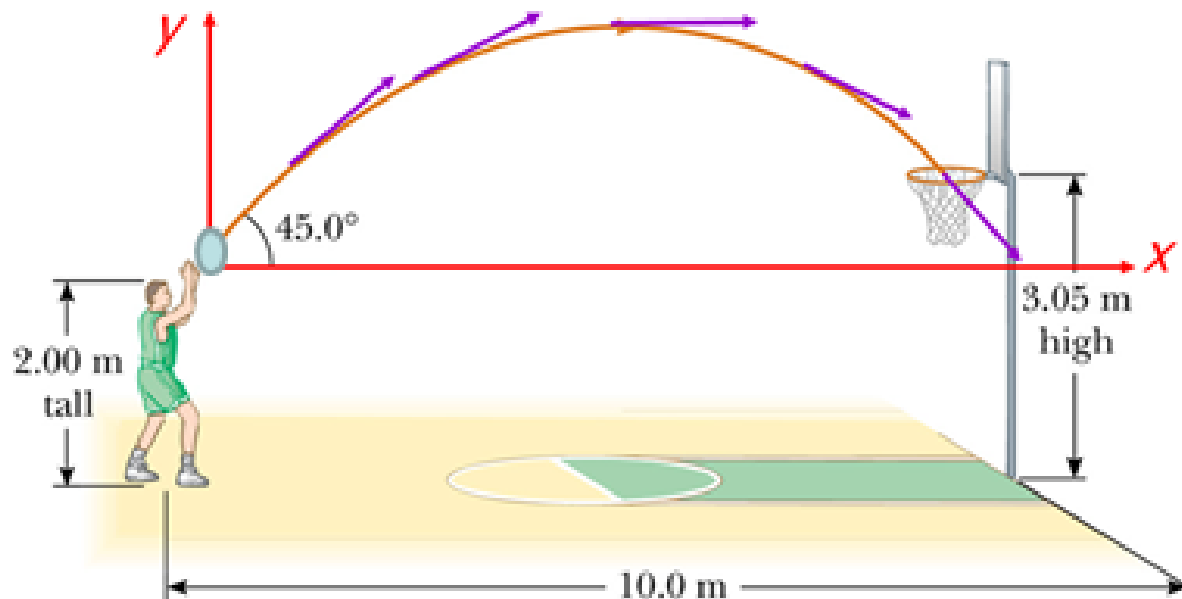
What are technical languages?

Answer:

- ▶ They are the extension of natural languages
- ▶ They are in the forms of symbols and equations, which describe the properties (statics), constraints (statics), and behaviors (dynamics) in the physical world for the sake of facilitating compact representations and precise computations manually or automatically.

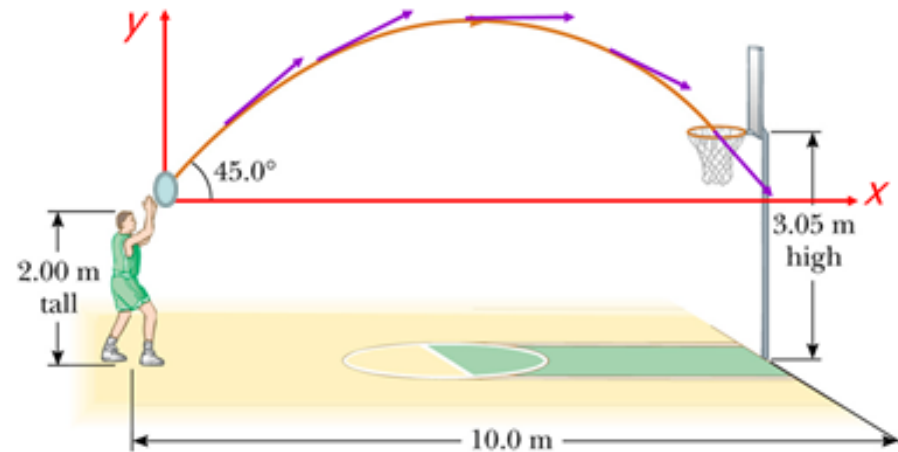
Example of Using Natural Language

- ▶ How to **literally** provide knowledge to the player for him/her to score the shooting of a basketball in the following scenario?



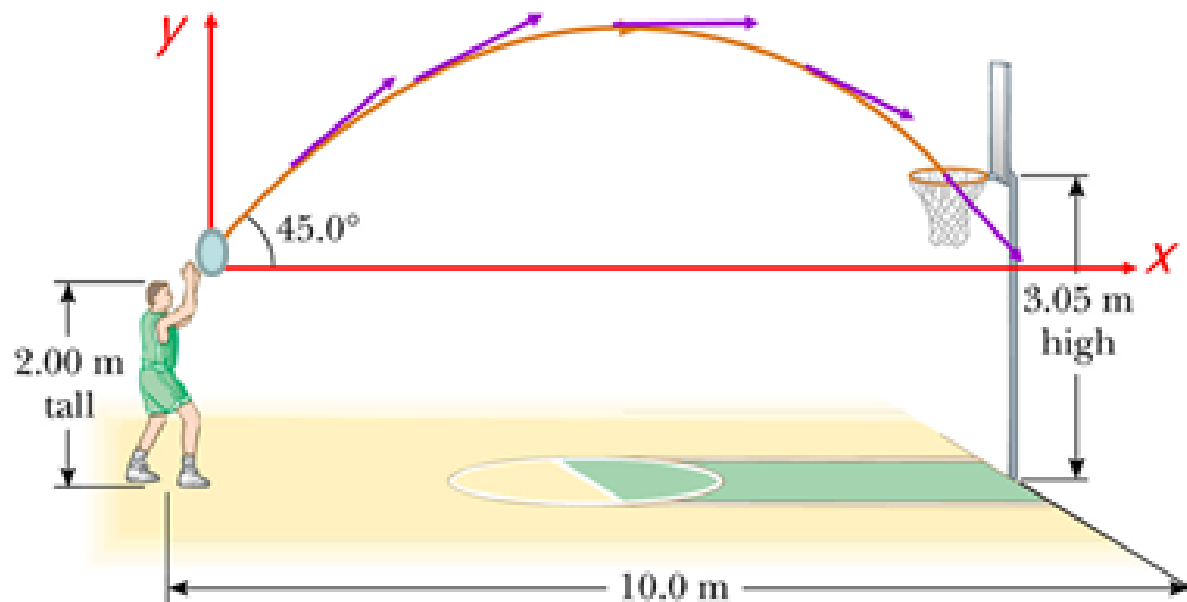
Answer

- ▶ To adjust your body so as to face the basket
- ▶ To make sure that you launch the ball in a vertical plane
- ▶ To launch the ball with an appropriate angle according to your experience
- ▶ To launch the ball with an appropriate force according to your experience



Example of Using Technical Language

- ▶ How to **scientifically** provide knowledge to the player for him/her to score the shooting of a basketball in the following scenario?



Answer

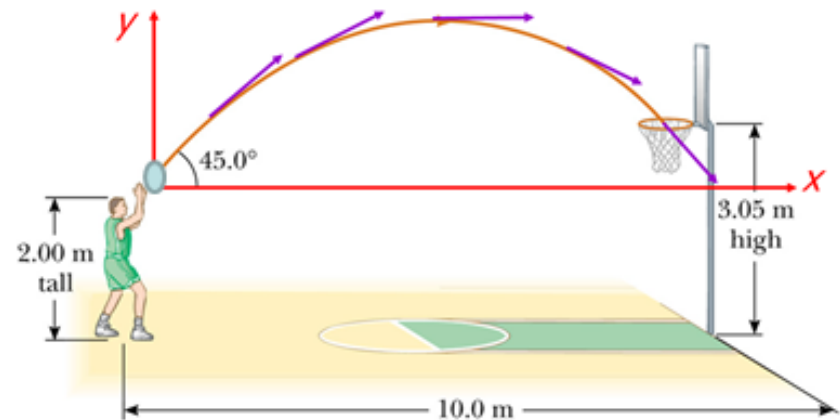
- ▶ To launch the ball at the height of 2.0 meters within a vertical plane.
- ▶ To launch the ball at the angle of 45.0 degrees with the initial speed of v_0 , which is calculated by using the following two equations:

$$\Delta y = 3.05 - 2.0 = v_0 \sin(45^\circ)t + \frac{1}{2}(-g)t^2$$

$$\Delta x = 10.0 = v_0 \cos(45^\circ)t$$

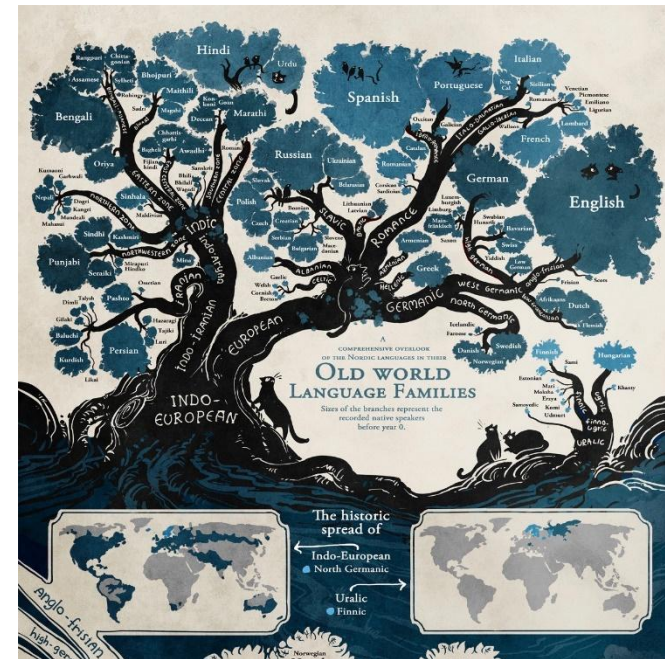
To apply force F_0 for 0.2 seconds to generate v_0 . F_0 is calculated from:

$$F_0 = \frac{mv_0}{0.2}$$



Outline

- ▶ Representation of Knowledge
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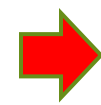


Invention of Human Languages

(a language is a virtual sensor which produces outputs of symbols)

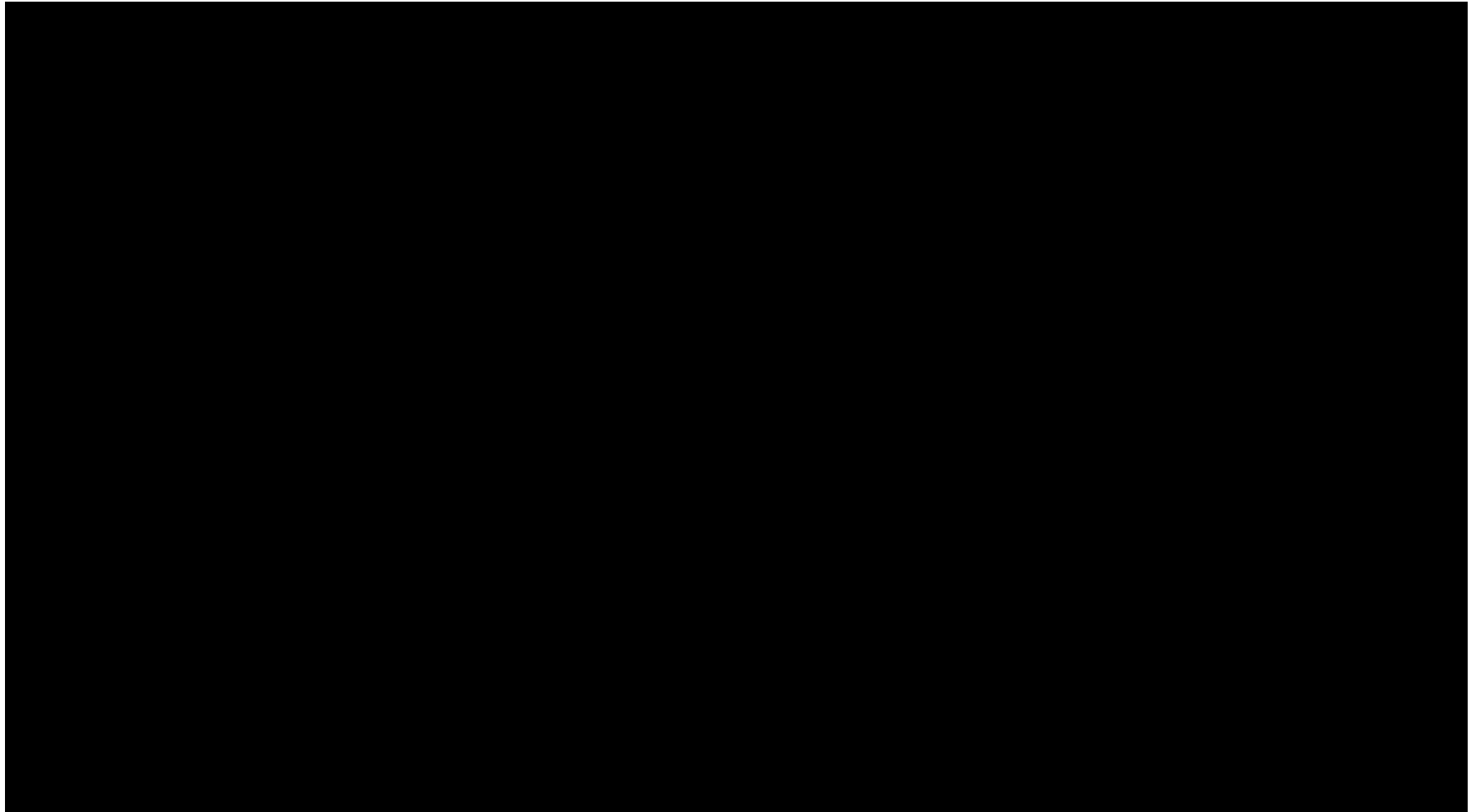
Why to invent programming languages?

- ▶ Knowledge could be perceived.
- ▶ Knowledge could be represented.
- ▶ Knowledge could be computed manually.
- ▶ Knowledge could be computed automatically.

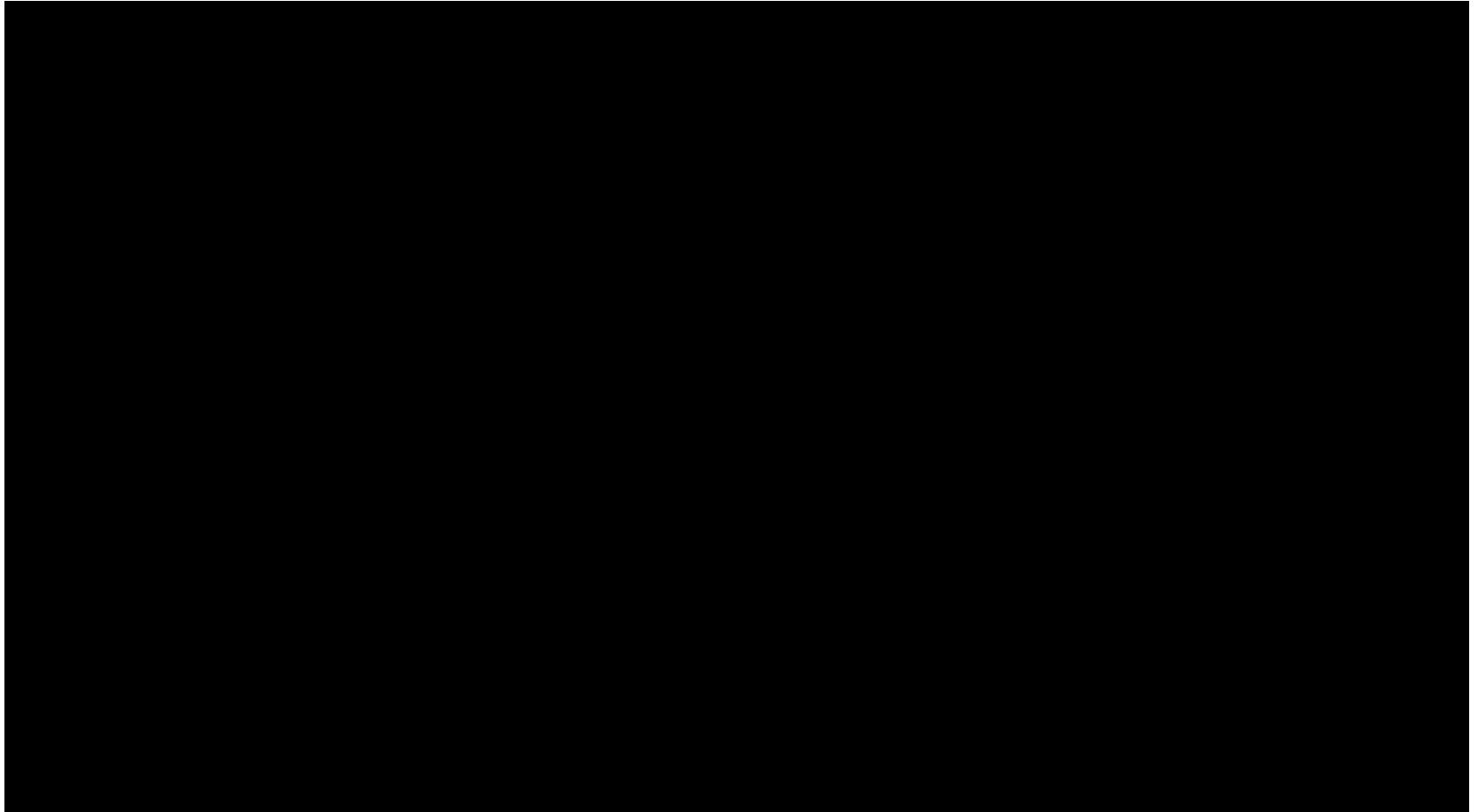


Next
Slide

Early History of Programming Languages ...

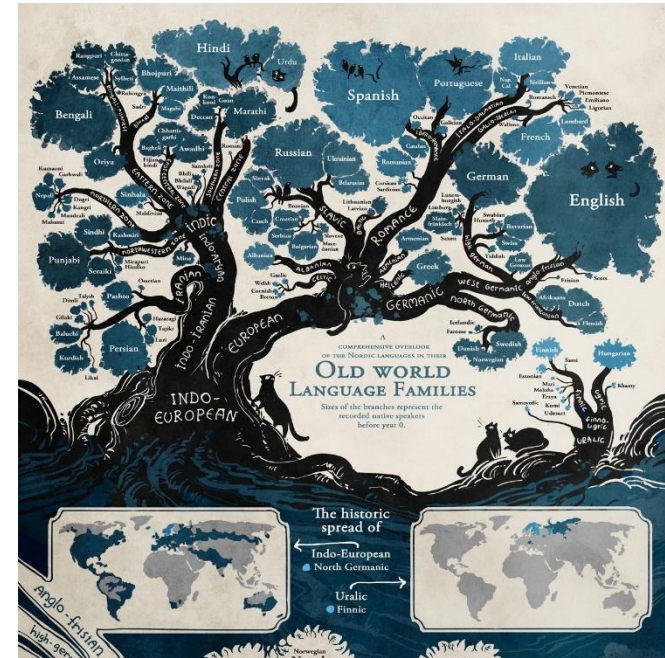


Popular Programming Languages ...



Summary

- ▶ Representation of Knowledge
- ▶ Definition of Conceptual Worlds
- ▶ Creation of Conceptual Worlds



- ▶ Invention of Natural Languages
- ▶ Invention of Technical Language
- ▶ Invention of Programming Languages

Invention of Human Languages

(a language is a virtual sensor which produces outputs of symbols)

Outline of Module 1

- ▶ Lecture 1:
 - ▶ Basics of Physical World
- ▶ Lecture 2:
 - ▶ Randomness of Physical World
- ▶ Lecture 3:
 - ▶ Basics of Conceptual Worlds
- ▶ Lecture 4:
 - ▶ Fuzziness of Conceptual Worlds





NANYANG
TECHNOLOGICAL
UNIVERSITY

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 1 Lecture 4

MA4822

Fuzziness of Conceptual Worlds

Xie Ming, PhD (France)

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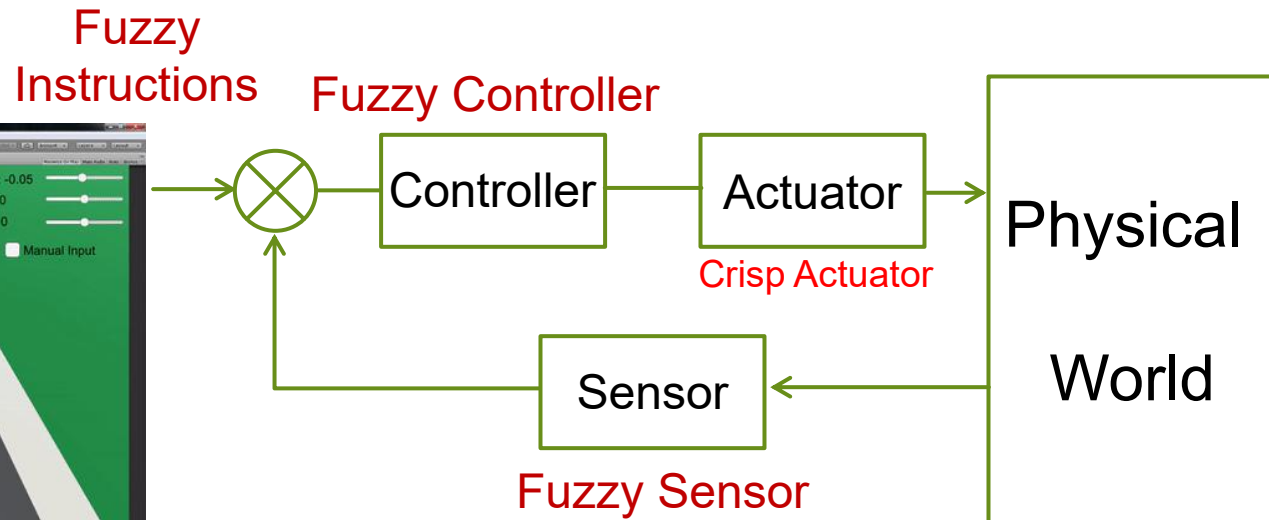
<http://personal.ntu.edu.sg/mmxie>

Outline

- ▶ Background Knowledge
 - ▶ Fuzziness of Natural Languages
 - ▶ Concept of Belief's Fuzzy Sets
 - ▶ Concept of Action's Fuzzy Sets
-
- ▶ Applications

Adaptation to variation of sensory data is an important capability of human intelligence!

Part of Large Knowledge Model



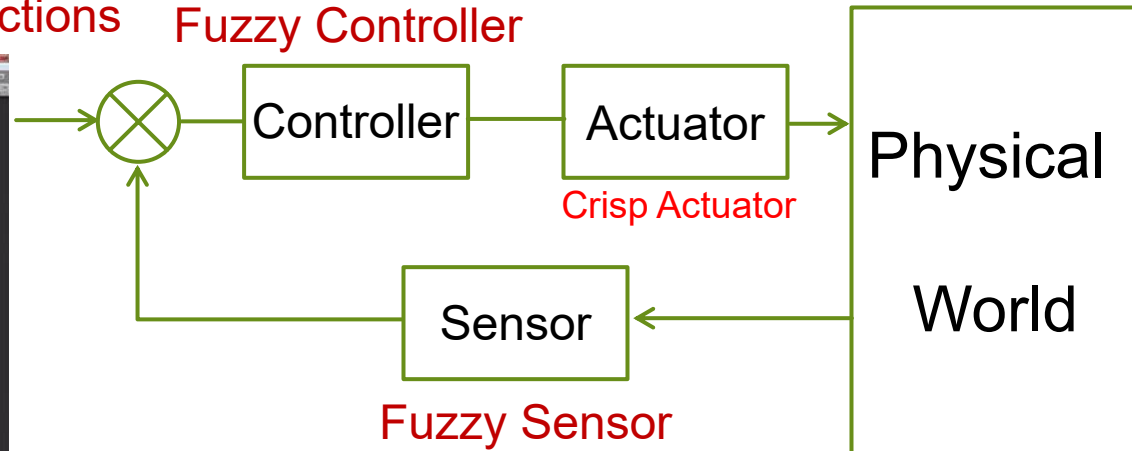
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} Part of Large Knowledge Model

Fuzzy Instructions Fuzzy Controller



History of Fuzzy Theory

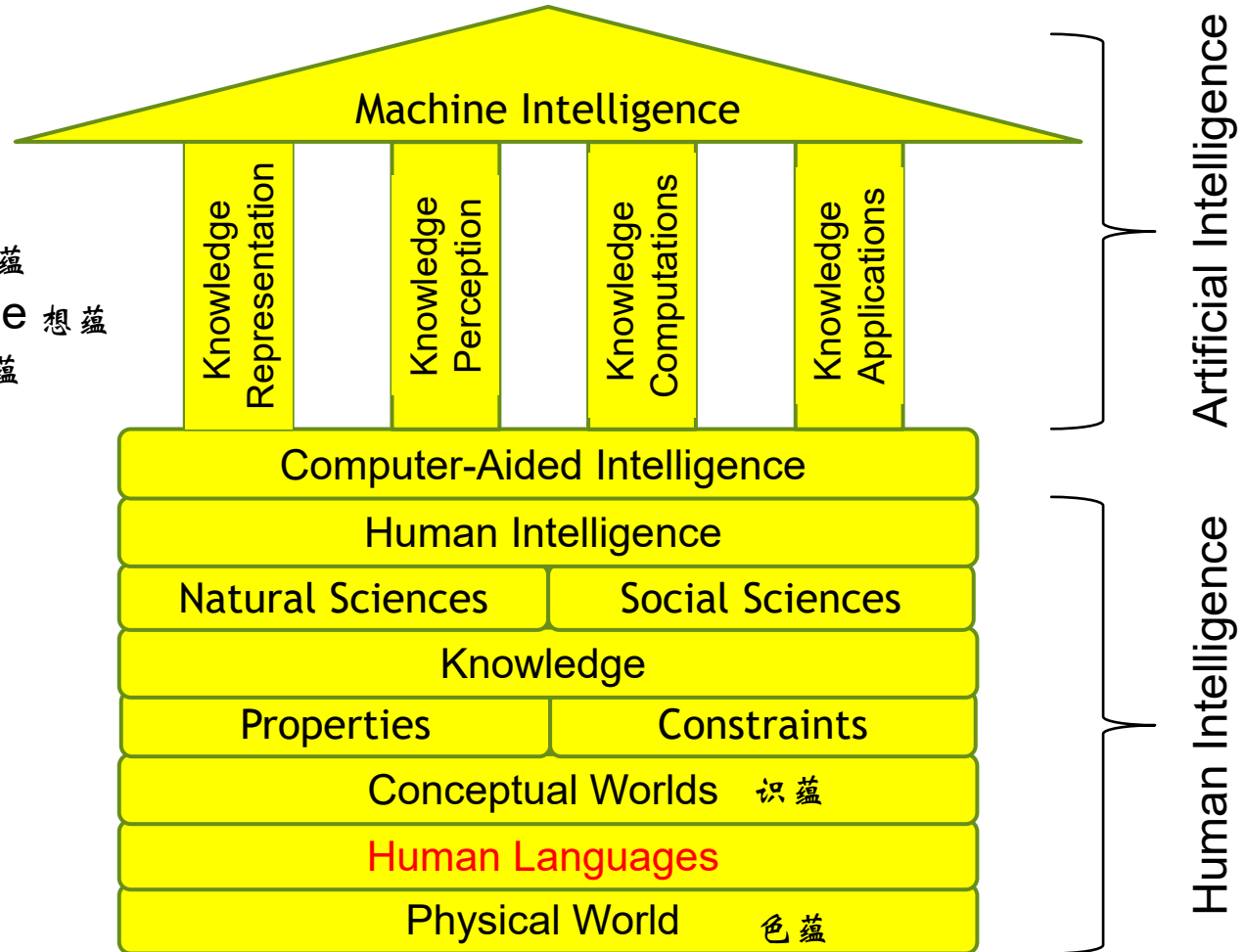
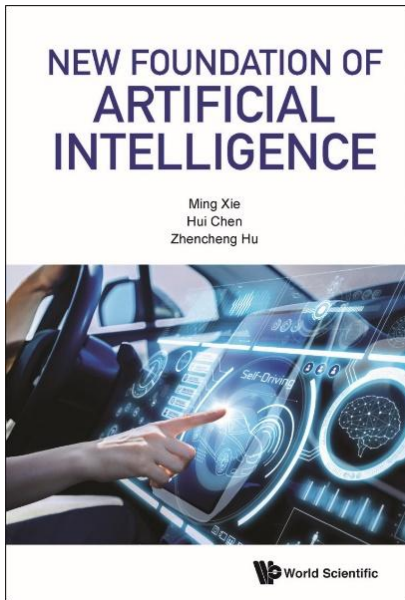
- ▶ Fuzzy logic had, however, been studied since the 1920s, as **infinite-valued logic**—notably by **Lukasiewicz** and **Tarski**.
- ▶ The term *fuzzy logic* was introduced with the 1965 proposal of **fuzzy set theory** by scientist **Lotfi Zadeh**.
- ▶ Many of the early successful applications of fuzzy logic were implemented in Japan.
- ▶ The first notable application was on the subway train in **Sendai**, in which fuzzy logic was able to improve the economy, comfort, and precision of the ride.

MIMO = Sum of MISOs = Sum of SIMOs

Landscape of Authentic AI ... (MA4829)

- One Tool
- Two Worlds
- Three Intelligences
- Four Pillars

- Signal to Knowledge 受 蕴
- Knowledge to Knowledge 想 蕴
- Knowledge to Signal 行 蕴

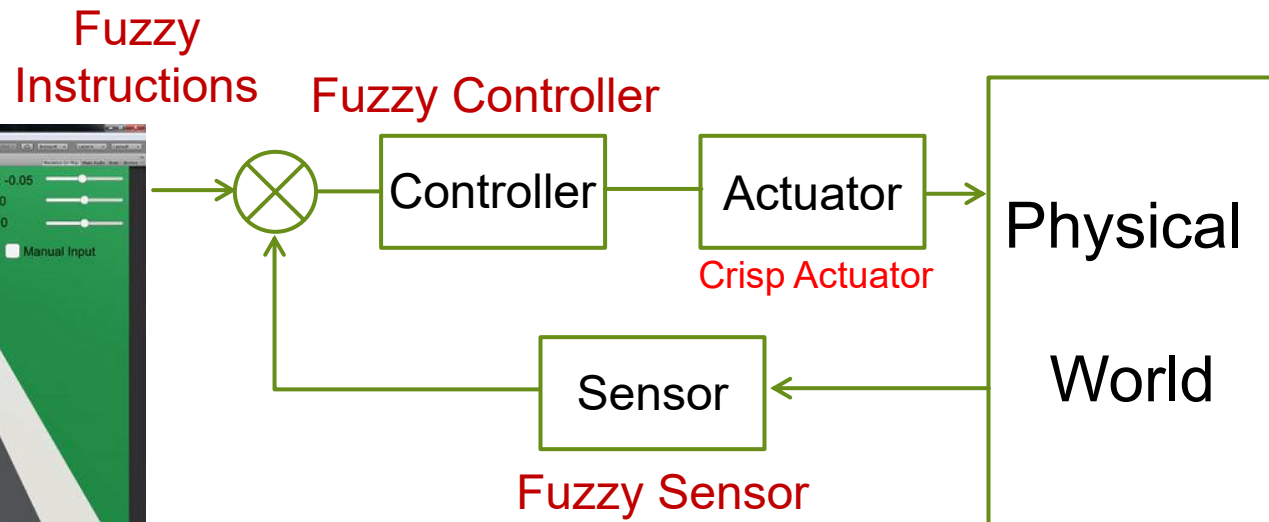


Outline

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Part of Large Knowledge Model



What is The Nature of Human Languages?

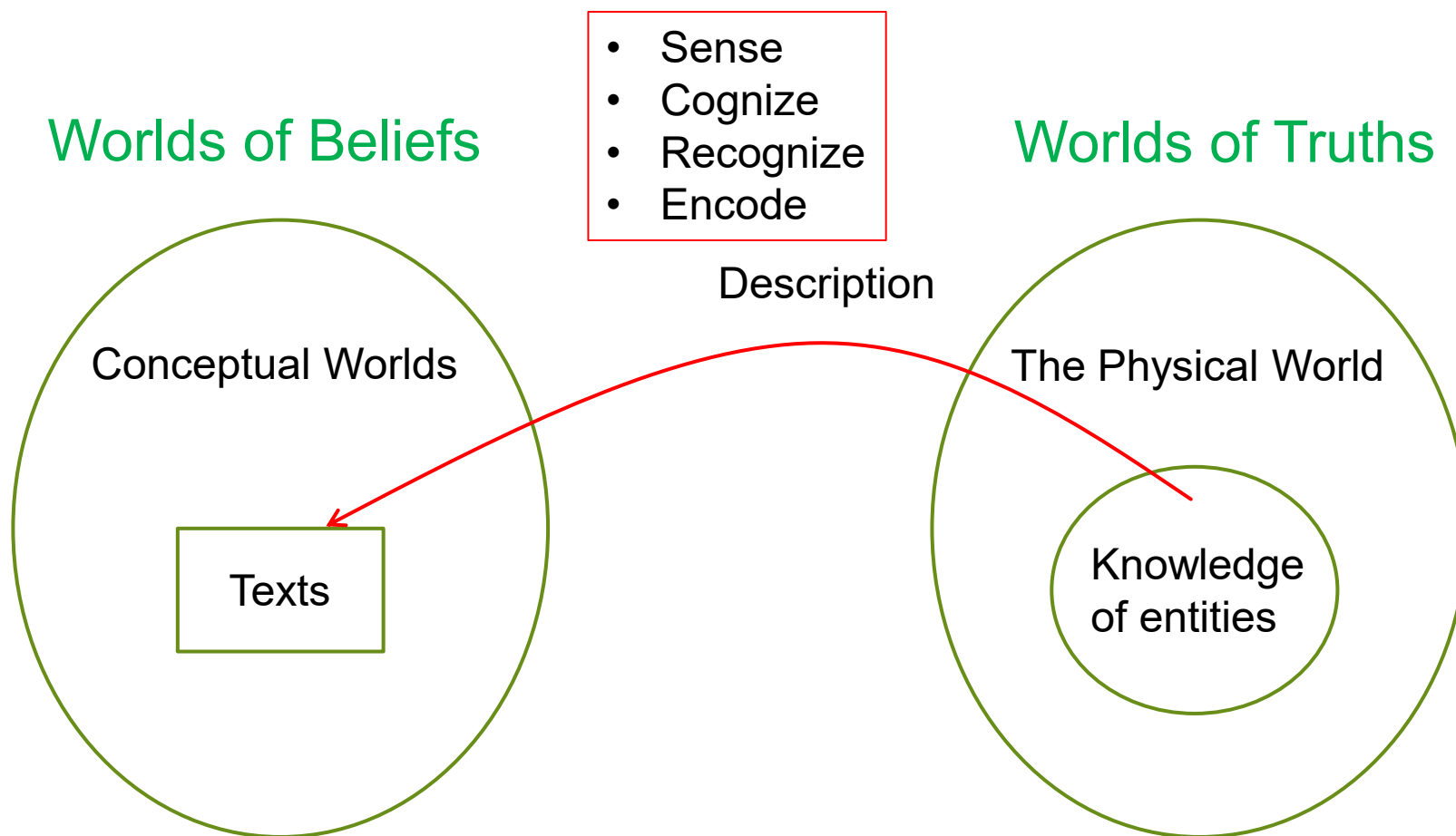
- ▶ Human languages are tools for our sensory systems to **encode** knowledge **perceived** from the physical world.
- ▶ The outcome of such **encoding** are texts in conceptual worlds.
- ▶ Texts in **conceptual worlds** enable us to reconstruct (i.e., **decoding**) the knowledge in the **physical world**.

There are Two Purposes for Human Beings to Use Human Languages

- ▶ **Knowledge Description** (via Perception)
 - ▶ From physical world to conceptual worlds
- ▶ **Knowledge Understanding** (via Reconstruction)
 - ▶ From conceptual worlds to physical world

MIMO = Sum of MISOs = Sum of SIMOs

Knowledge Description: Illustration



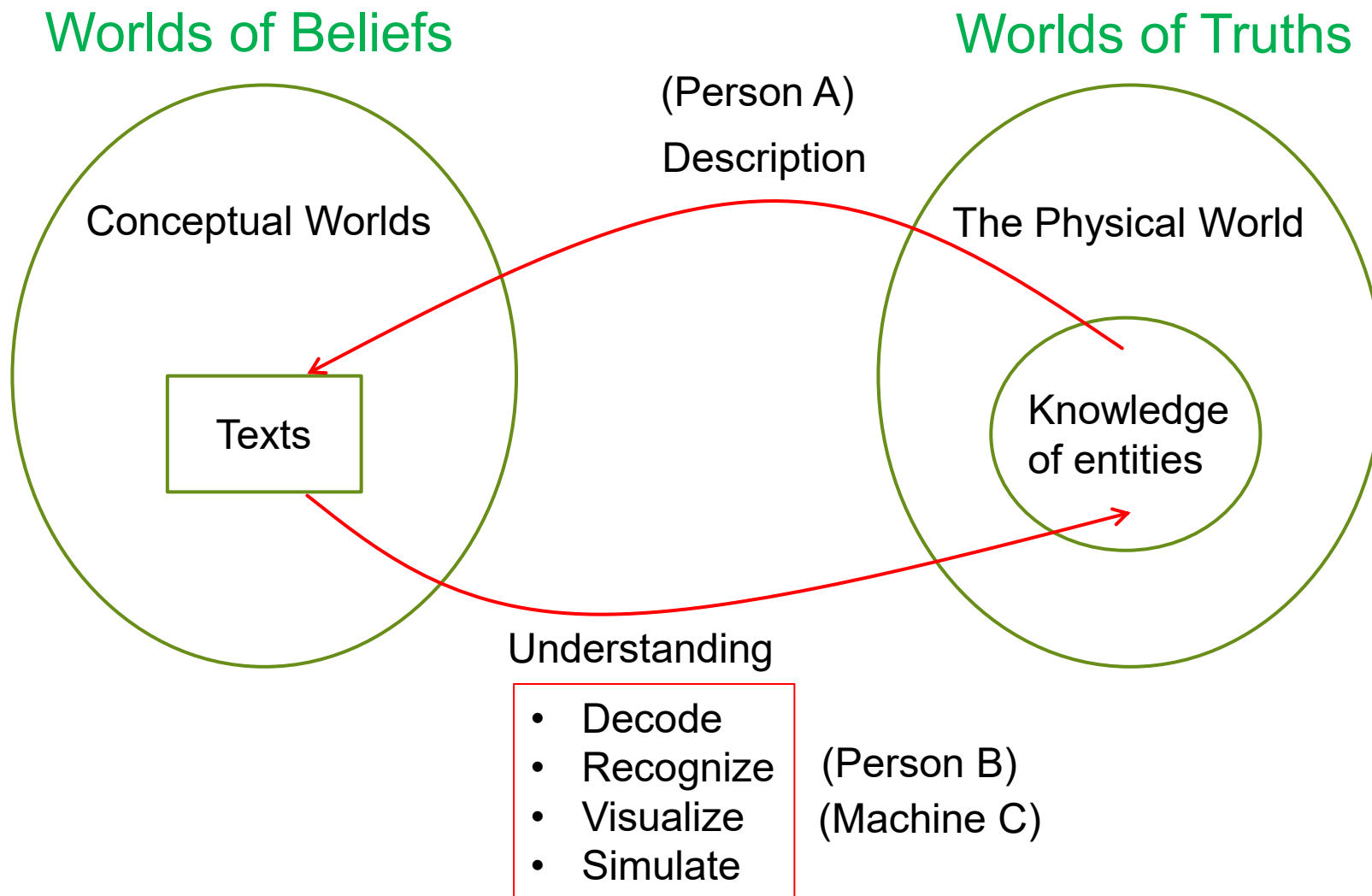
We are more affected by Beliefs than by Truths!

Process of Knowledge Description

It involves two steps:

- ▶ Use of sensory systems to sense, cognize, and recognize, the knowledge.
- ▶ Use of words, phrases and sentences to encode the cognized, or recognized, knowledge.

Knowledge Understanding: Illustration



Process of Knowledge Understanding

It involves two steps:

- ▶ To decode, or recognize, knowledge from texts.
- ▶ To visualize, or simulate, the recognized knowledge in the physical world.

In fact, the sensory systems of human beings could output accurate data to a certain extent ... (this is an advantage)

▶ A car is moving.

▶ A car is following another car.

▶ A car stops at a junction.

▶ There is a pedestrian.

Car's Size?

Car's Speed?

Car's Weight?

It seems that our sensory systems intentionally produce fuzzy outputs ... (this is an advantage)

Example 1



- ▶ A car is moving very fast.
- ▶ A car is following another car closely.
- ▶ That car is beautiful and large.
- ▶ This car is powerful.

Example 2

- ▶ That truck is heavily loaded.
- ▶ That car is very noisy.
- ▶ The street is very crowded.
- ▶ The street-light is too dim.

Discussion: Is fuzziness about belief or vagueness?

Challenges Faced by Authentic AI

- ▶ All knowledge in the physical world are crisp.
- ▶ Not all knowledge in conceptual worlds are crisp (NOTE: this is an advantage).
- ▶ **Challenge 1: (Pillar No.2 of AI)**
 - ▶ How to transform crisp sensory data into fuzzy sensory beliefs or conceptual knowledge?
 Use of Belief's Fuzzy Sets
- ▶ **Challenge 2: (Pillar No.4 of AI)**
 - ▶ How to transform fuzzy linguistic instructions into crisp signals for action-taking or control?
 Use of Action's Fuzzy Sets

Example of Fuzzy Set Associated with Belief “Tall Man”

- ▶ A tall man may refer to a man of the height of 1.80 m.
- ▶ A tall man may refer to a man of the height of 1.85 m
- ▶ A tall man may refer to a man of the height of 1.90 m



{A tall man} \longleftrightarrow {Infinite Number of Instances of Height}

This is an advantage of doing knowledge compression!

Example of Fuzzy Set Associated with Belief “Red Color”

- ▶ The sensory data (255, 0, 0) could trigger the belief of red color.
- ▶ The sensory data (255, 1, 0) could also trigger the belief of red color.
- ▶ The sensory data (255, 1, 1) could also trigger the belief of red color.

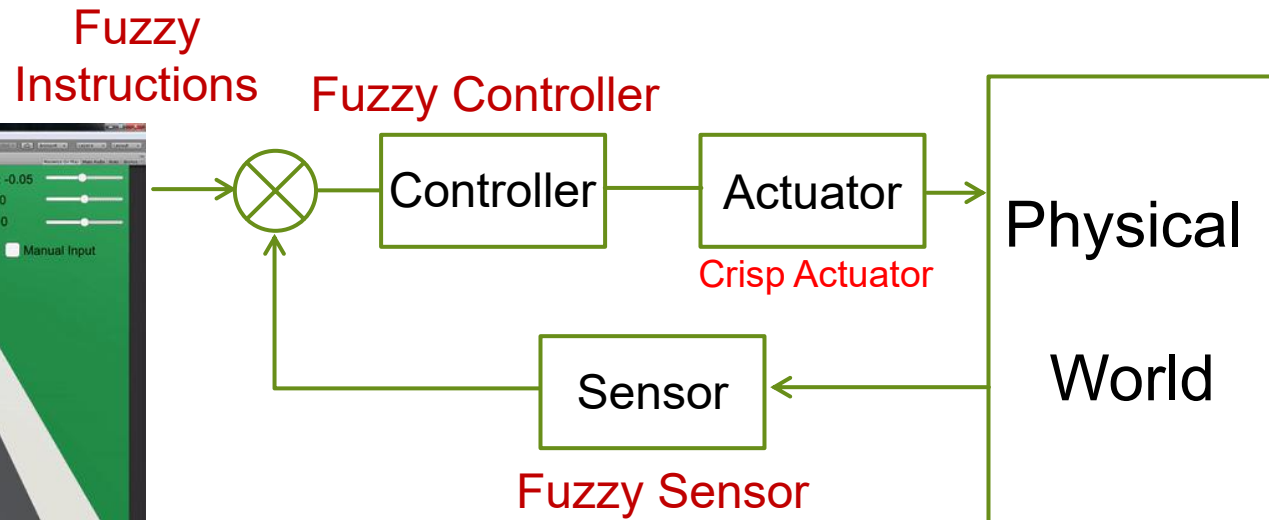


Outline

- ▶ Background Knowledge
- ▶ Fuzziness of Natural Languages
- ▶ Concept of Belief's Fuzzy Sets
- ▶ Concept of Action's Fuzzy Sets
- ▶ Applications

Adaptation to variation of sensory data is an important capability of human intelligence!

Part of Large Knowledge Model



Important Remarks

- ▶ The sensory systems of human beings could not **output crisp data**.
- ▶ The sensory systems of human beings **output sensory beliefs**.

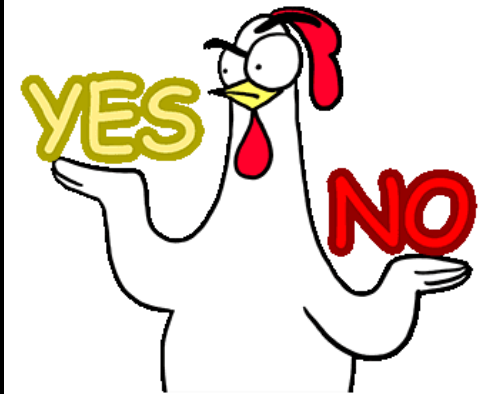
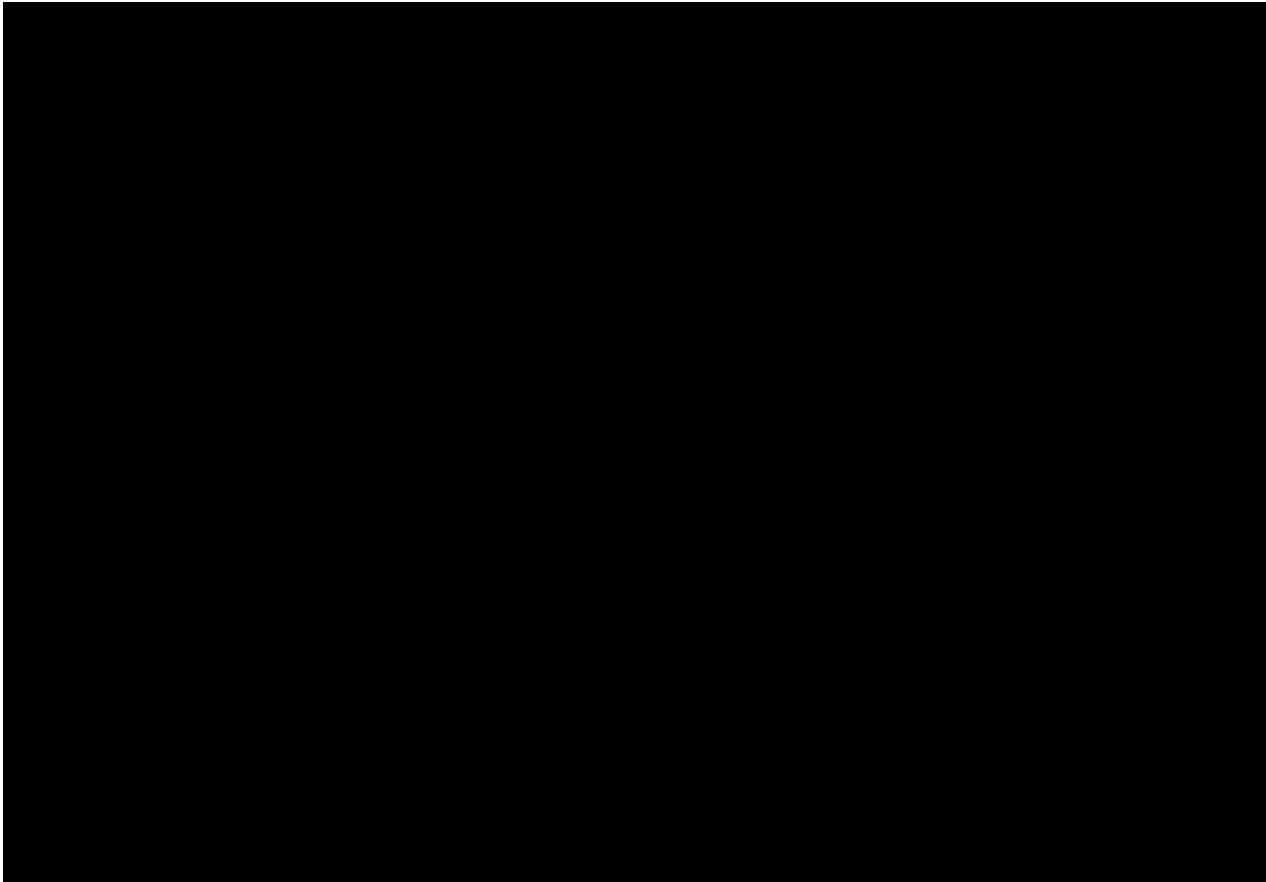


Issue of
Fuzzification

Issue of
Defuzzification

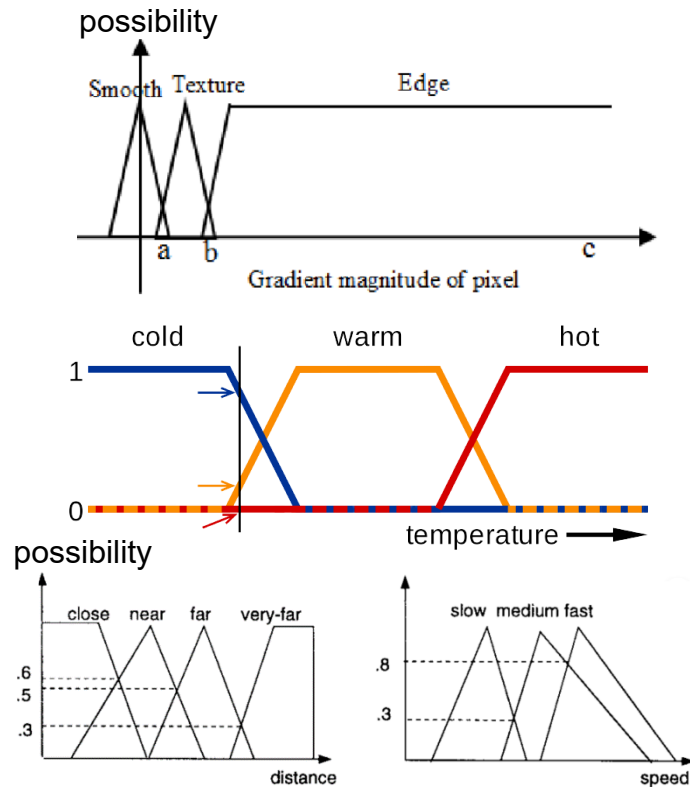
MIMO = Sum of MISO

Belief will affect us more than truth



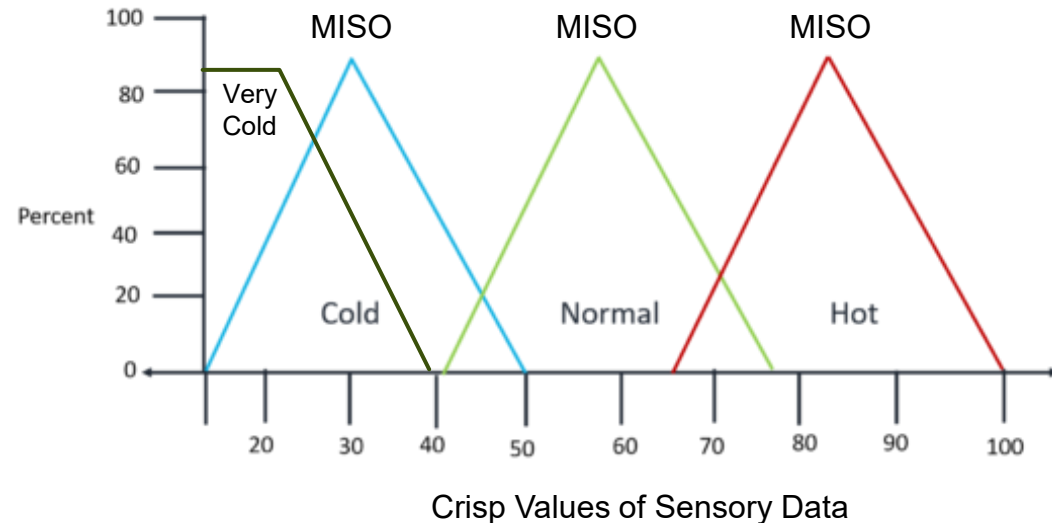
What is a sensory belief which is fuzzy?

- It refers to a specific knowledge in terms of properties, constraints or behaviors, which is associated with a set of sensory data with certain degrees of belief. Such association is one-to-many association. One belief correspond to a set of sensory data (i.e., MISO).



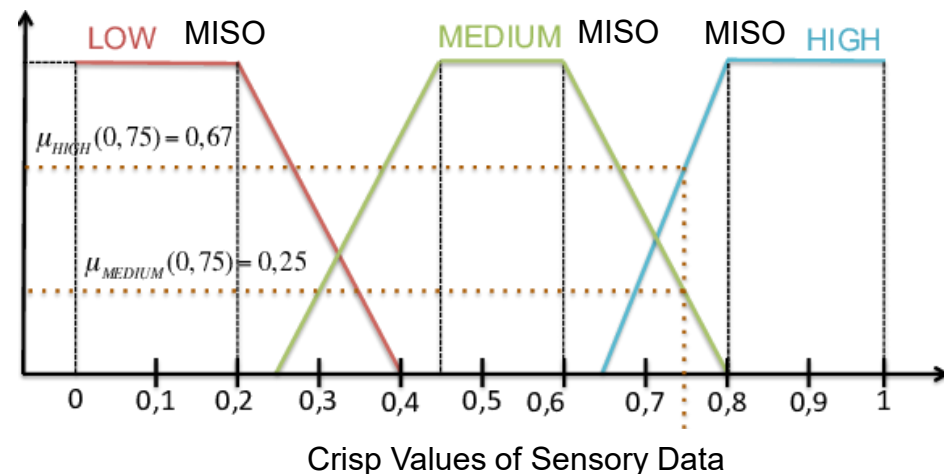
What is a belief's fuzzy set?

- ▶ All the sensory data, which could trigger a same belief with different levels of possibility, consist of a **belief's fuzzy set**.



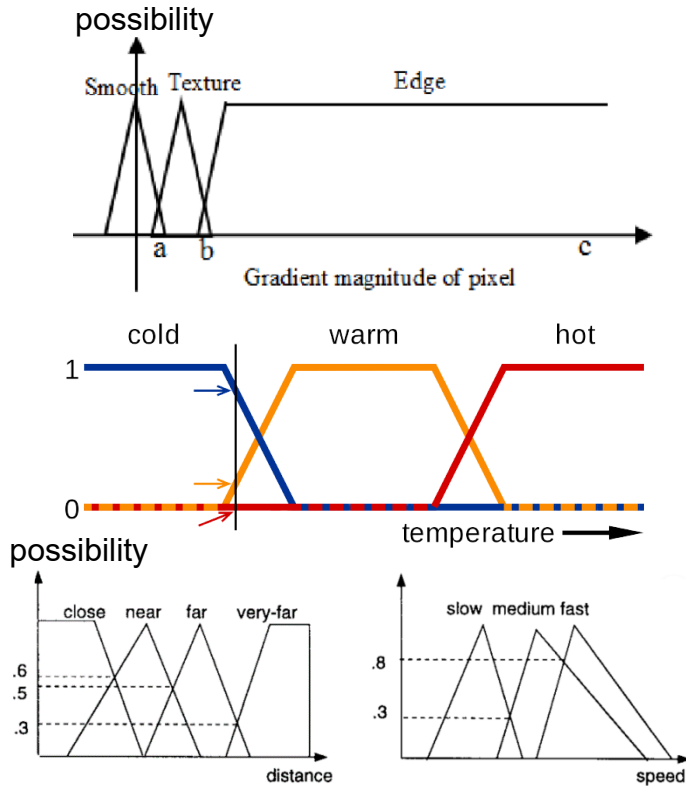
- ▶ Hence, a belief's fuzzy set corresponds to a set of sensory data.

$$\text{MIMO} = \text{Sum of MISO}$$



Definition of Possibility Function

- ▶ It is a function which takes values between 0 and 1.
- ▶ In fuzzification, such values indicate the possibilities for a set of sensory data to trigger a fuzzy belief about specific knowledge (1st type of output from AI systems).
- ▶ In defuzzification, such values reveal the possibilities for a fuzzy action to produce a set of crisp values for specific behaviors (2nd type of output from AI systems).



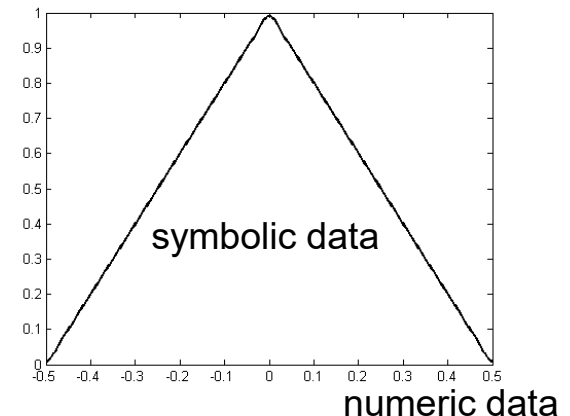
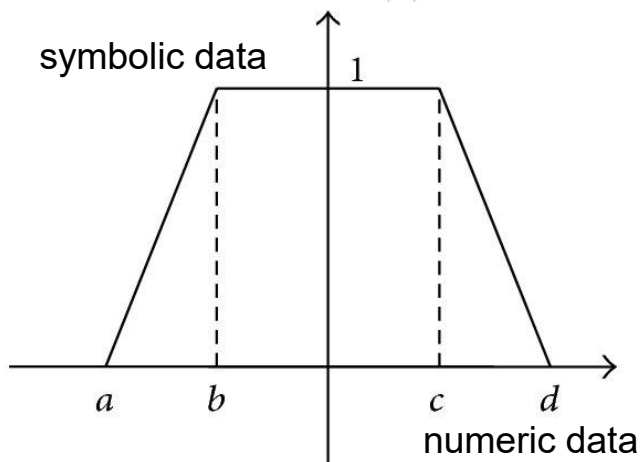
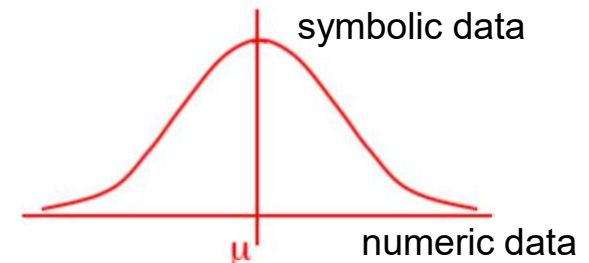
How to determine possibility functions?

- Possibility functions such as normalized Gaussian functions

$$P(x) = e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Possibility functions such as trapezoidal functions

- Possibility functions such as triangular functions



Representation of Belief's Fuzzy Set

- ▶ Given a set of n crisp sensory data (input):

$$X = \{x_i, i = 1, 2, 3, \dots, n\}$$

- ▶ Given a set of m fuzzy beliefs (output) and their possibility functions:

$$B = \{(b_j, \mu_j(x_i)), i = 1, 2, 3, \dots, n; j = 1, 2, 3, \dots, m\}$$

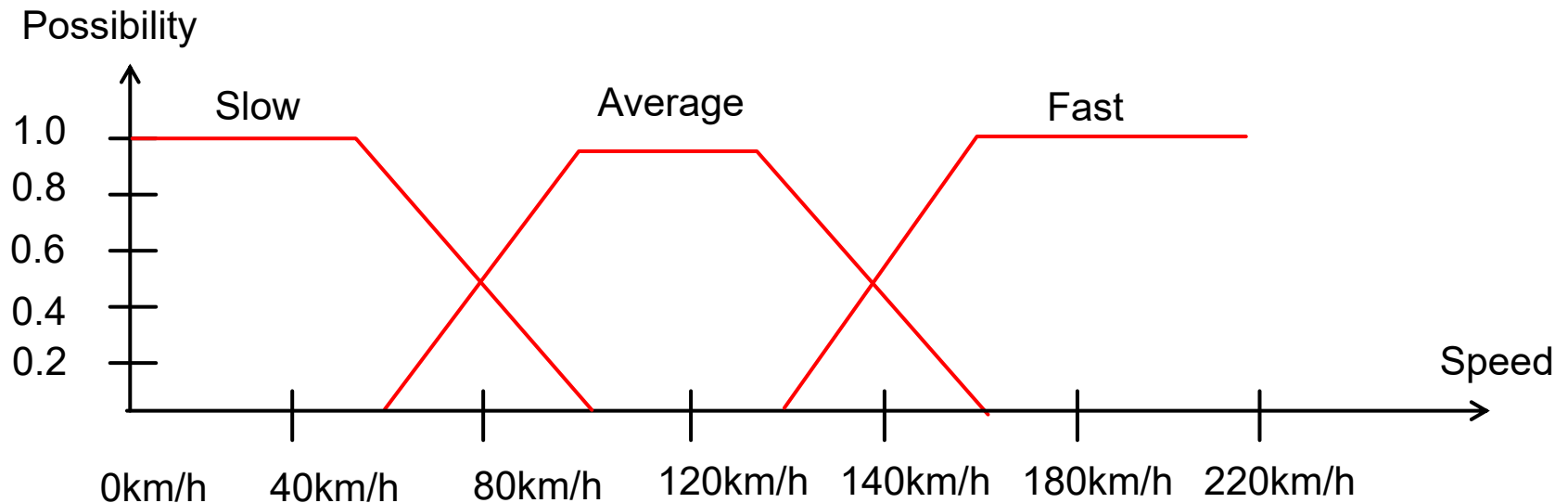
- ▶ Then, the fuzzy set associated with j -th fuzzy belief is represented as:

$$F_B(b_j) = \{(x_i, \mu_j(x_i)), i = 1, 2, \dots, n\}, j = 1, 2, 3, \dots, m$$

MIMO = Sum of MISO

Example of Fuzzy Sets Associated with Beliefs

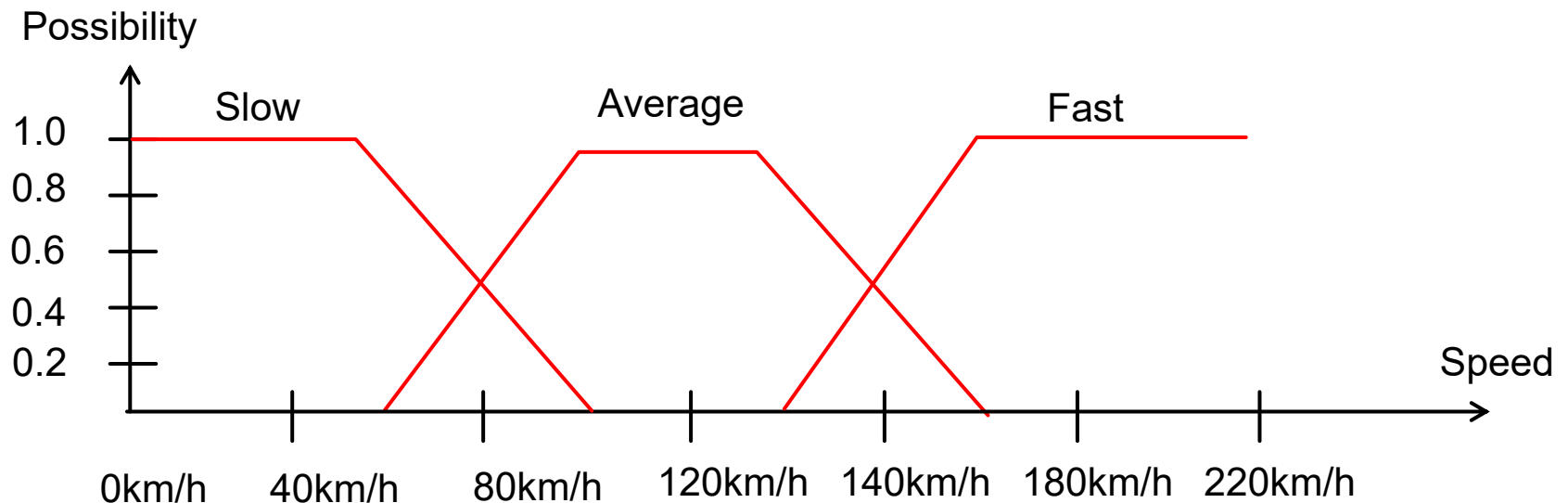
- ▶ As shown in the figure below, what are the three fuzzy sets associated with the three fuzzy beliefs if the speed's sensory data set is {40, 80, 120, 140, 180, 220} (km/h)?



MIMO = Sum of MISO

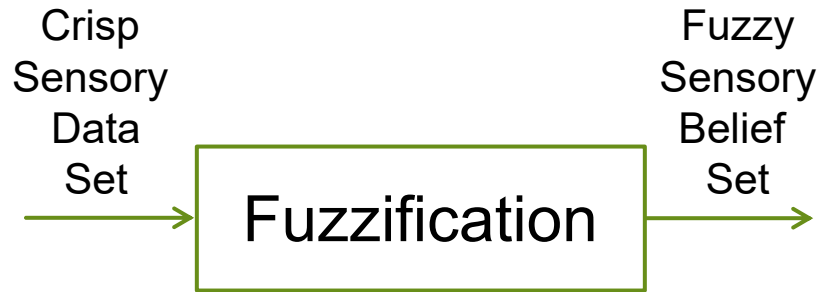
Answer

- ▶ $F_B(\text{Slow}) = \{(40, 1.0), (80, 0.5), (120, 0), (140, 0), (180, 0), (220, 0)\}$
- ▶ $F_B(\text{Normal}) = \{(40, 0), (80, 0.5), (120, 1.0), (140, 0.5), (180, 0), (220, 0)\}$
- ▶ $F_B(\text{Fast}) = \{(40, 0), (80, 0), (120, 0), (140, 0.5), (180, 1.0), (220, 1.0)\}$



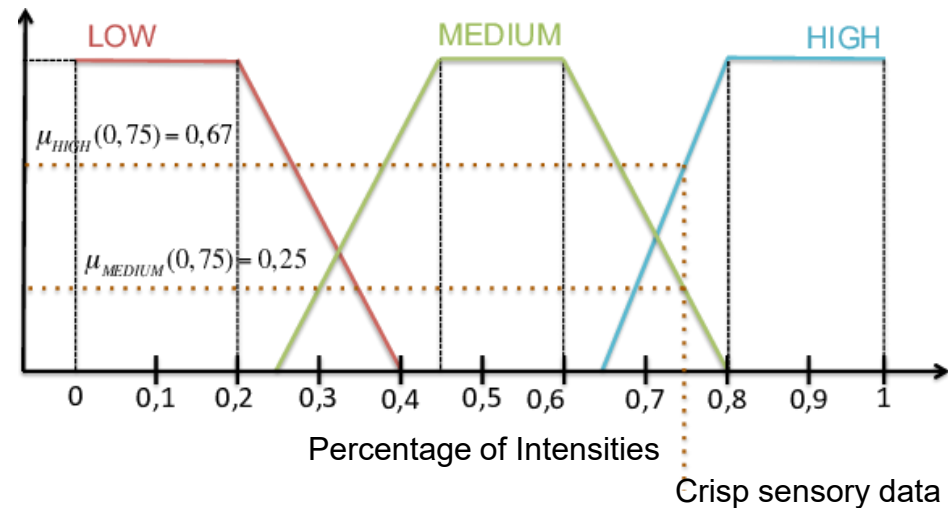
Applications of Fuzzy Sets Associated with Beliefs

- ▶ Transformation of sensory data into sensory beliefs.
- ▶ Solution to the Symbol Grounding Problem in AI.



MIMO = Sum of MISO

Image Brightness



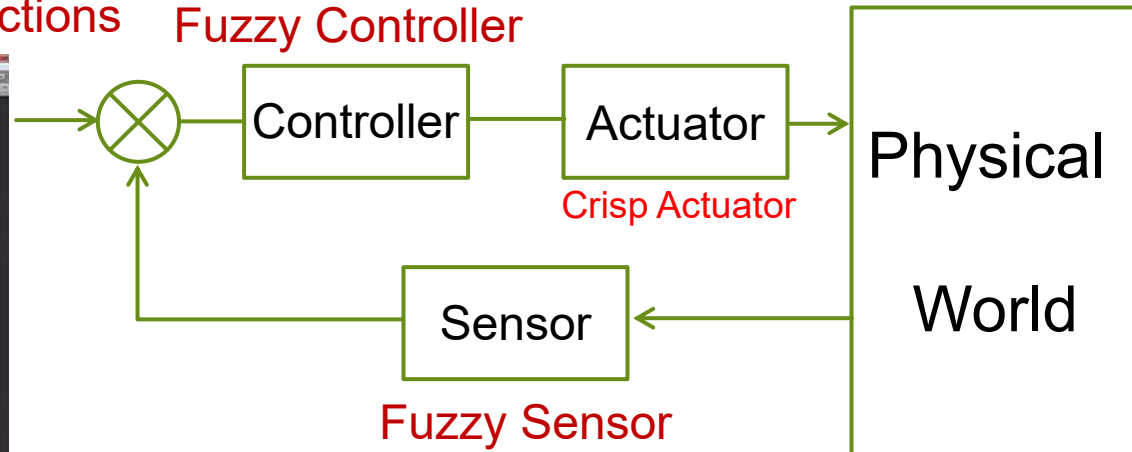
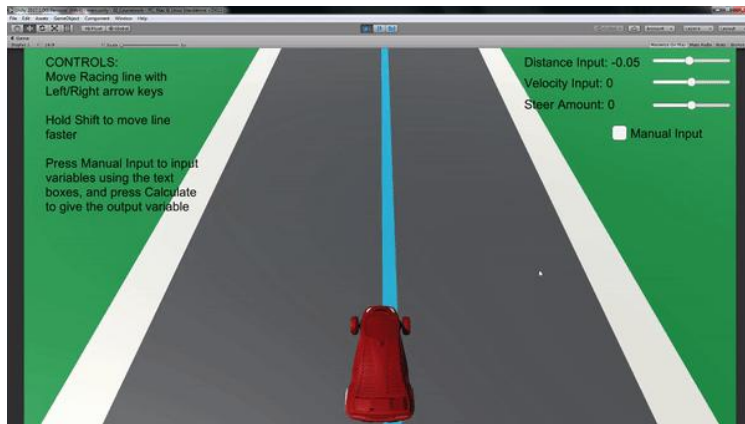
Outline

- ▶ Background Knowledge
 - ▶ Fuzziness of Natural Languages
 - ▶ Concept of Belief's Fuzzy Sets
 - ▶ Concept of Action's Fuzzy Sets
-
- ▶ Applications

Adaptation to variation of sensory data is an important capability of human intelligence!

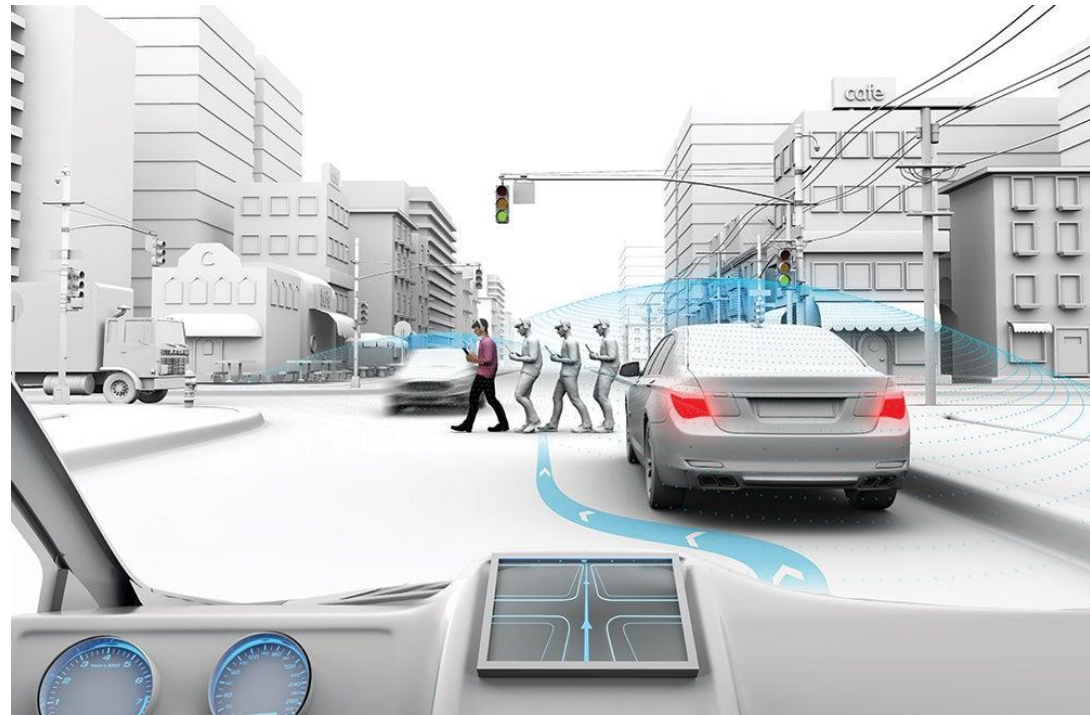
Part of Large Knowledge Model

Fuzzy Instructions Fuzzy Controller



Important Remarks

- ▶ Human beings are skillful with the use of sensory beliefs in order to **discover knowledge**.
- ▶ Human beings are also skillful with the use of sensory beliefs in order to **determine crisp values of control signal** for action-taking.



If fuzzy conditions, then fuzzy decisions of actions.

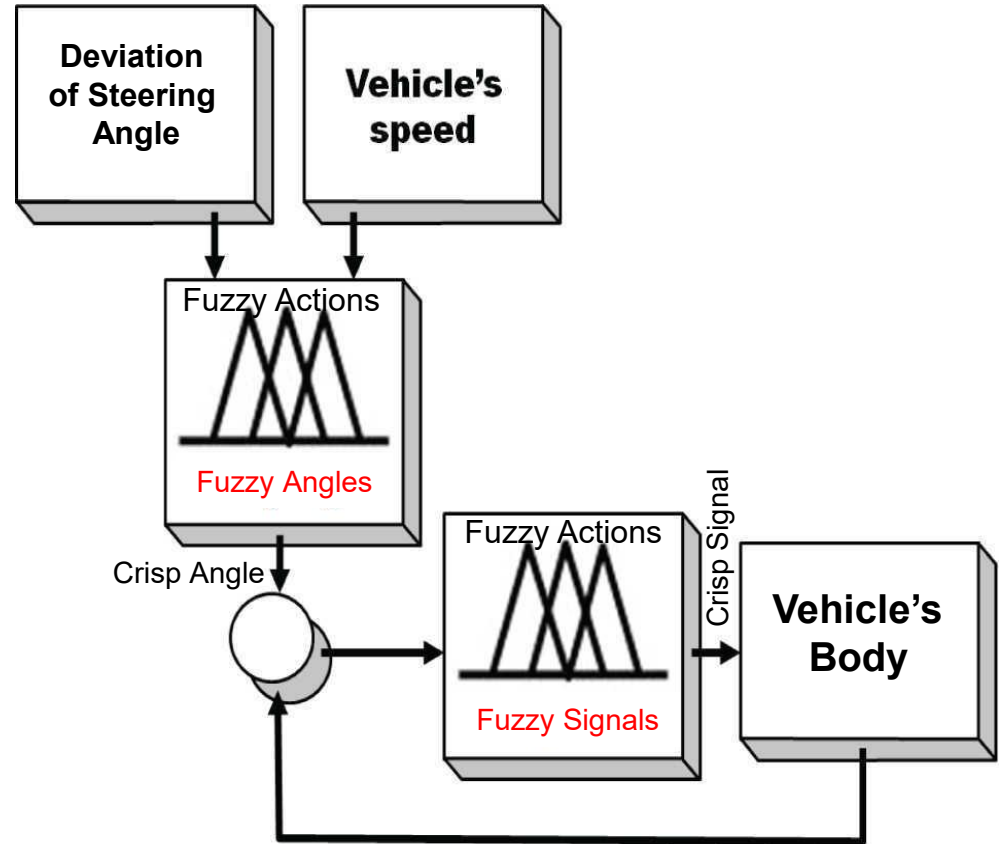
If fuzzy decisions, then crisp values for action-taking.

Question to Focus on ...

- ▶ How to transform **fuzzy decisions** of actions into **crisp values** of actions which will enable signal-driven **controls**?

- ▶ Answer:

- ▶ Use of Action's Fuzzy Sets



From Fuzzy Decisions of Actions to Fuzzy Sets

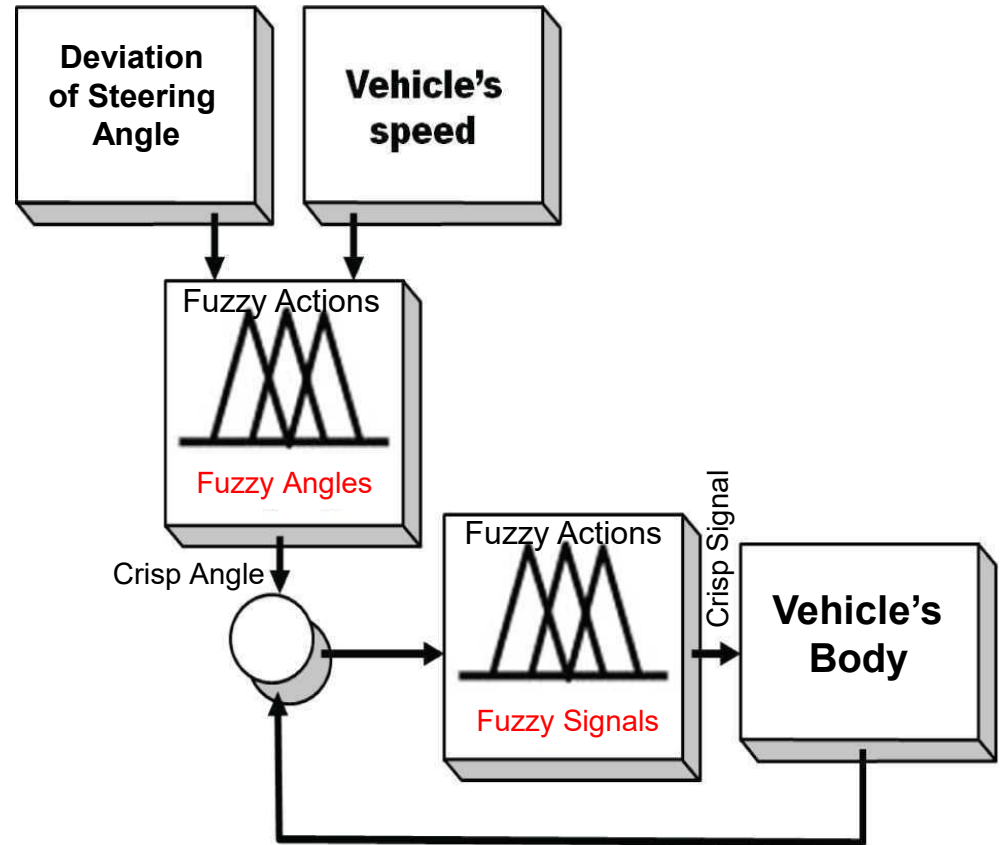
From Fuzzy Sets to Crisp Values of Actions

What is an action's fuzzy set?

- ▶ An action's fuzzy set is a set of **fuzzy decisions** of action associated with a single **crisp value** of action which enable action-taking or control.



MIMO = Sum of MISO



From Fuzzy Decisions of Actions to Fuzzy Sets

From Fuzzy Sets to Crisp Values of Actions

Example of Guessing an Action's Fuzzy Set ...

- ▶ A car's speed is measured to be 140 km/h. Then, this **crisp data** of action may reveal **fuzzy decisions** of action such as:
 - ▶ Person A may say: "I am 60% certain that the car has received the fuzzy command which asks it to move at a normal speed".
 - ▶ Person B may say: "I am 90% certain that the car has received the fuzzy command which asks it to move a quite fast speed".
- ▶ A car's speed is measured to be 50 km/h. Then, this **crisp data** of action may reveal **fuzzy decisions** of action such as:
 - ▶ Person C may say: "I am 40% certain that the car has received the fuzzy command which asks it to move slowly".
 - ▶ Person D may say: "I am 70% certain that the car has received the fuzzy command which asks it to move at a normal speed".

Representation of Action's Fuzzy Set

- ▶ Given a set of n crisp data (output) and m fuzzy decisions (input):

$$Y = \{y_i, i = 1, 2, 3, \dots, n\} \quad D = \{d_j, j = 1, 2, 3, \dots, m\}$$

- ▶ The possibility function associated with j -th fuzzy decision is: $\mu_j(y_i)$

- ▶ The possibility value for j -th fuzzy decision to output i -th crisp data is:

$$P(d_j) = \mu_j(y_i)$$

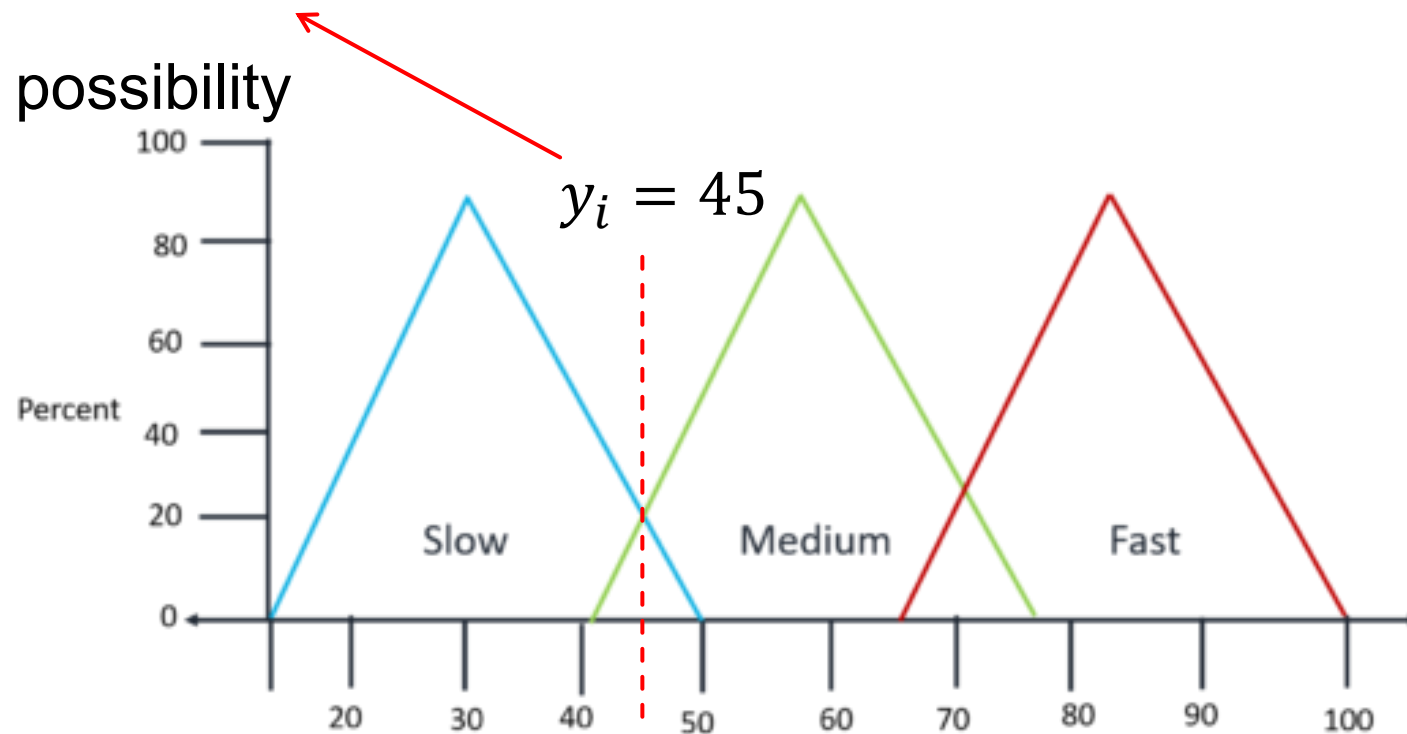
- ▶ Then, the fuzzy set associated with i -th crisp data is represented as:

$$F_A(y_i) = \{(d_j, \mu_j(y_i)), j = 1, 2, 3, \dots, m\}, i = 1, 2, 3, \dots, n$$

MIMO = Sum of MISO

Example of Action's Fuzzy Set for Vehicle's Speed Control ...

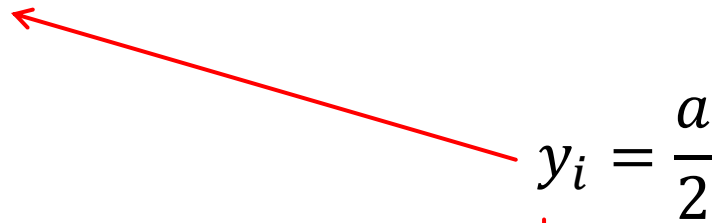
$$F_A(45) = \{(drive\ slowly, 0.2), (drive\ mediumly, 0.2), (drive\ fast, 0.0)\}$$



Crisp Values of Speed Control Data

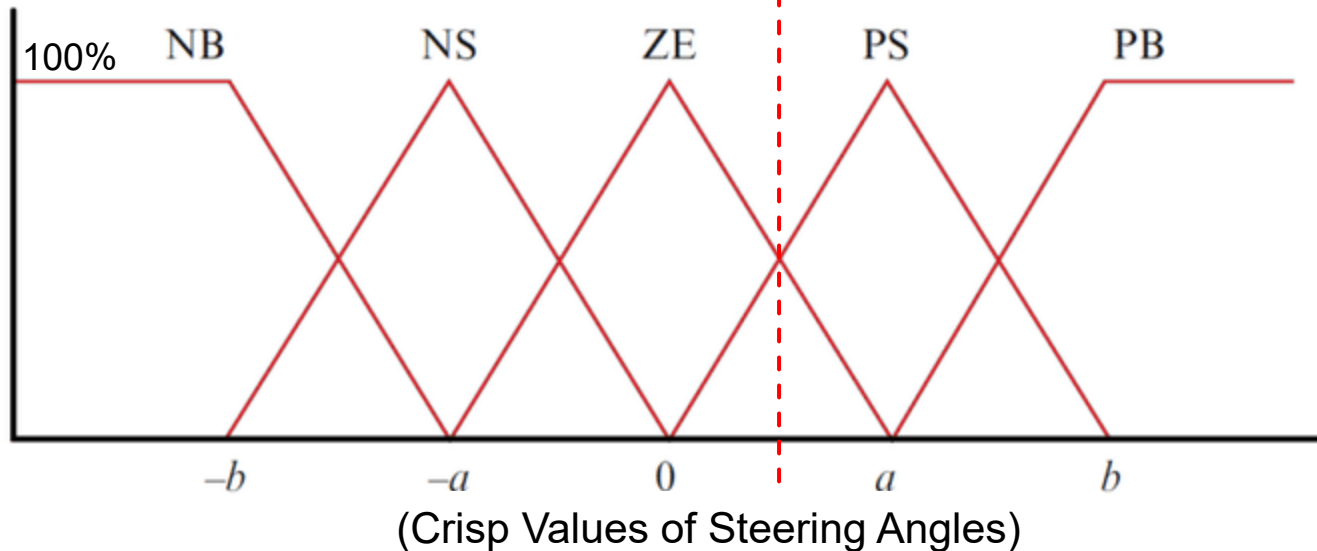
Example of Action's Fuzzy Set for Steering Angle Control ...

$$F_A(a/2) = \{(NB, 0.0), (NS, 0.0), (ZE, 0.5), (PS, 0.5), (PB, 0.0)\}$$



possibility

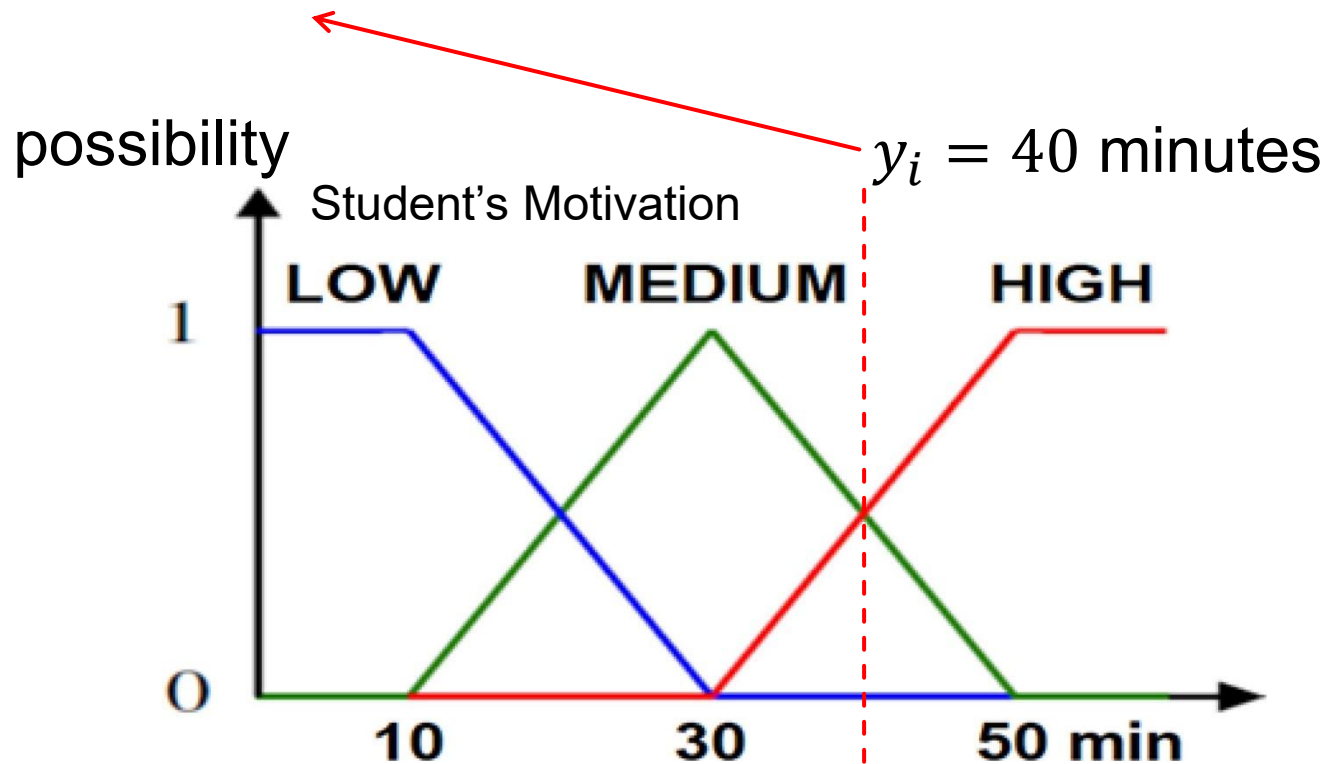
(fuzzy actions of steering)



- NB (Negative Big)*
- NS (Negative Small)*
- ZE (Zero)*
- PS (Positive Small)*
- PB (Positive Big)*

Example of Action's Fuzzy Set (i.e., Planning) for Daily Self-Study Time ...

$$F_A(40) = \{(low\ motivation, 0.0), (medium\ motivation, 0.5), (high\ motivation, 0.5)\}$$



Planning of Student's Self-Study Time per Day

Applications of Fuzzy Sets Associated with Beliefs and Actions ...

- ▶ To imitate human-like decision-making with fuzzy sensory data:
 - ▶ To transform crisp sensory data into fuzzy sets associated with beliefs
 - ▶ To apply fuzzy rules to determine fuzzy decisions
 - ▶ To transform fuzzy decisions into crisp decisions

AI's Pillar No. 3: knowledge to Knowledge



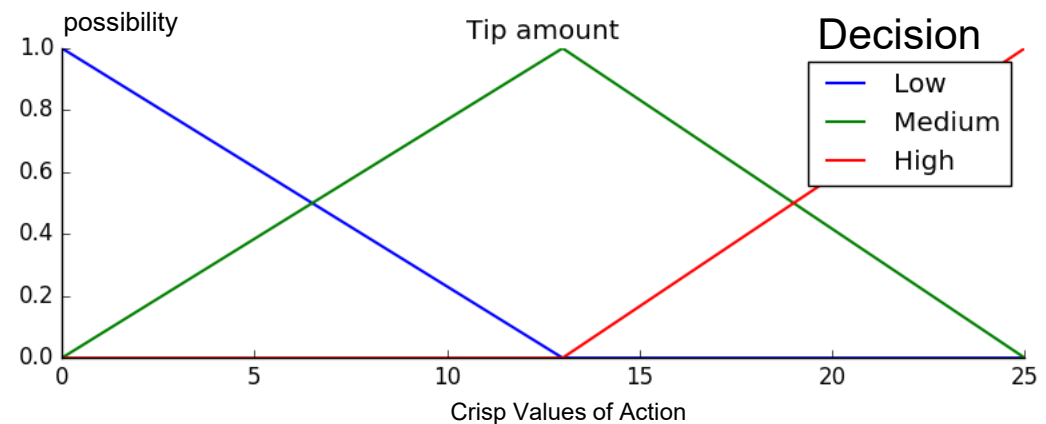
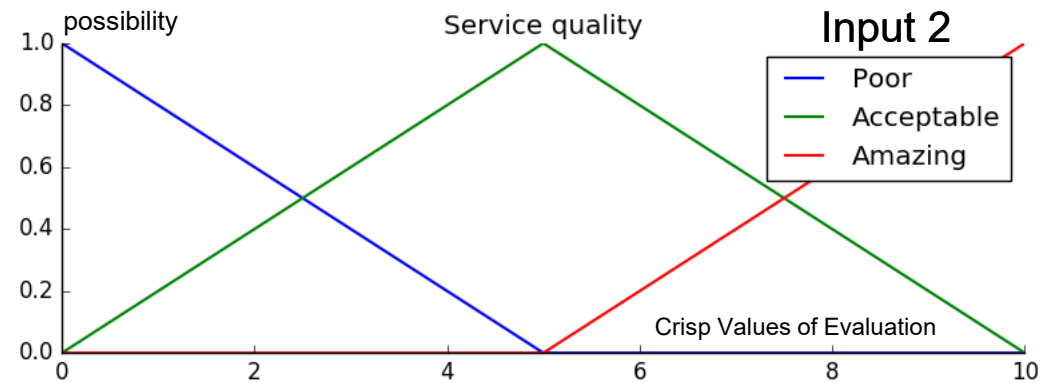
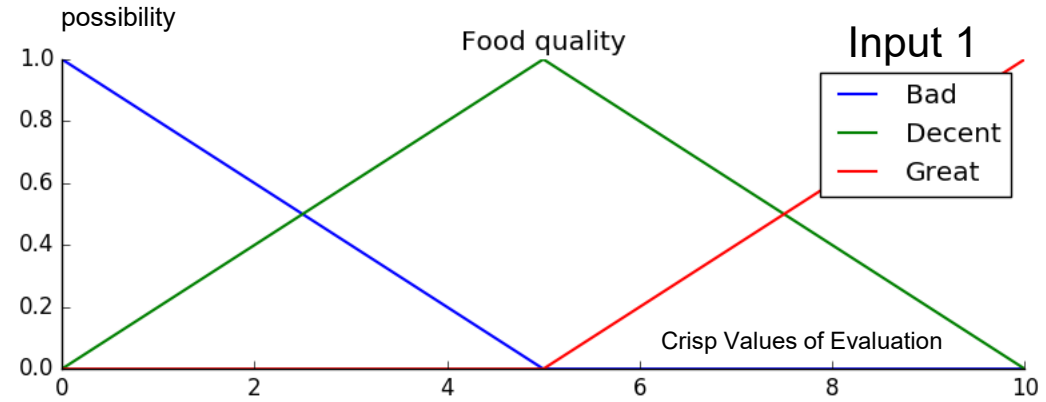
- ▶ To imitate human-like action-taking with fuzzy sensory data:
 - ▶ To transform crisp sensory data into fuzzy sets associated with beliefs
 - ▶ To apply fuzzy rules to determine fuzzy decisions of actions
 - ▶ To transform fuzzy decisions into crisp values for action-taking or control

MIMO = Sum of MISO

AI's Pillar No. 4: knowledge to Signal

Example of Imitating Human-Like Decision-Making in Restaurant ...

- Get evaluation of food quality
- Get evaluation of service quality
- Fuzzify both evaluation results
- If food quality is ..., then tip is ...
- If service quality is ..., then tip is ...
- Defuzzify the fuzzy decisions.
- Determine the crisp action.



Operations with Belief/Action's Fuzzy Sets:

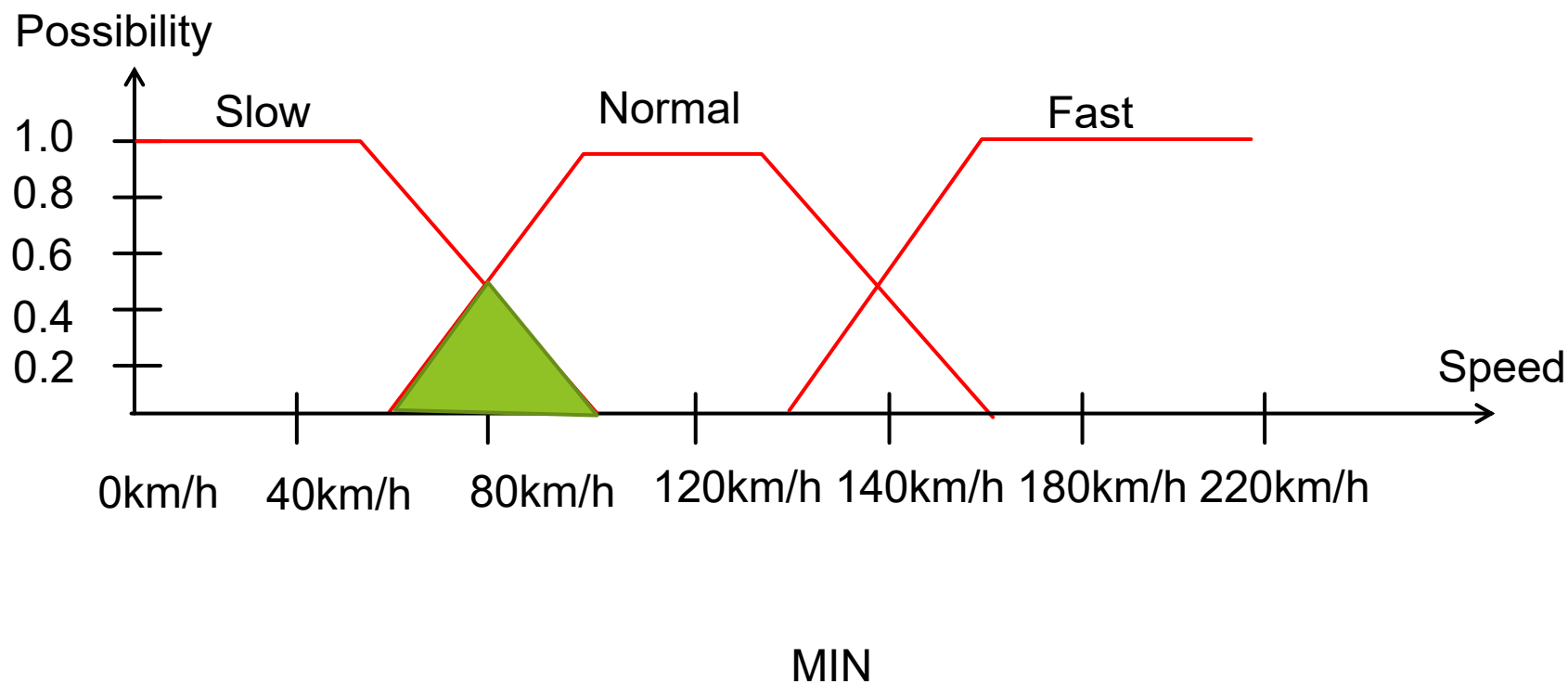
Conjunction of Two Fuzzy Sets ...

- It is the operation of intersection between two possibility functions within a sensor or a system.

$$\mu_{A \cap B}(x) = \mu_A(x) \cap \mu_B(x)$$

Example

- The conjunction of fuzzy set associated with belief “slow” and fuzzy set associated with belief “normal” is:



Operations with Belief/Action's Fuzzy Sets:

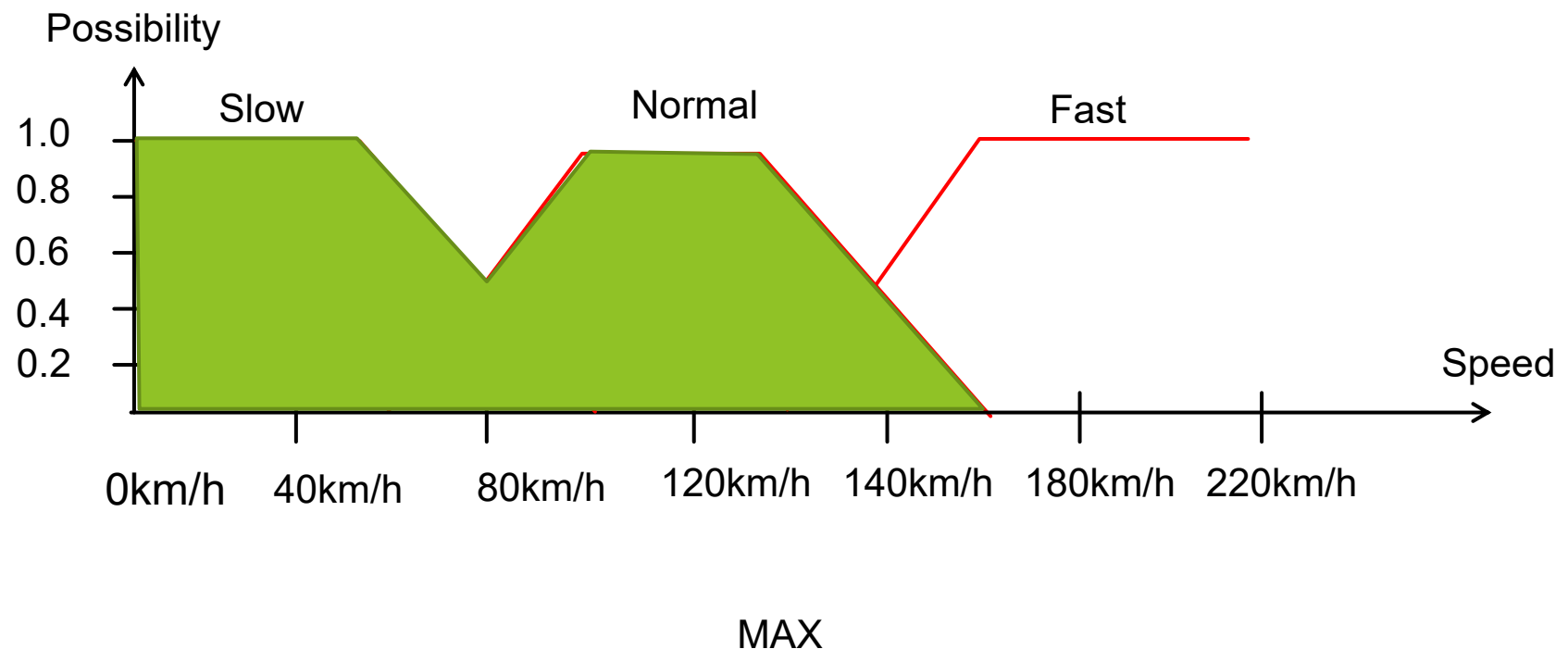
Disjunction of Two Fuzzy Sets ...

- ▶ It is the operation of union between two possibility functions within a sensor or a system.

$$\mu_{A \cup B}(x) = \mu_A(x) \cup \mu_B(x)$$

Example

- ▶ The disjunction of fuzzy set associated with belief “slow” and fuzzy set associated with belief “normal” is:



Operations with Belief/Action's Fuzzy Sets:

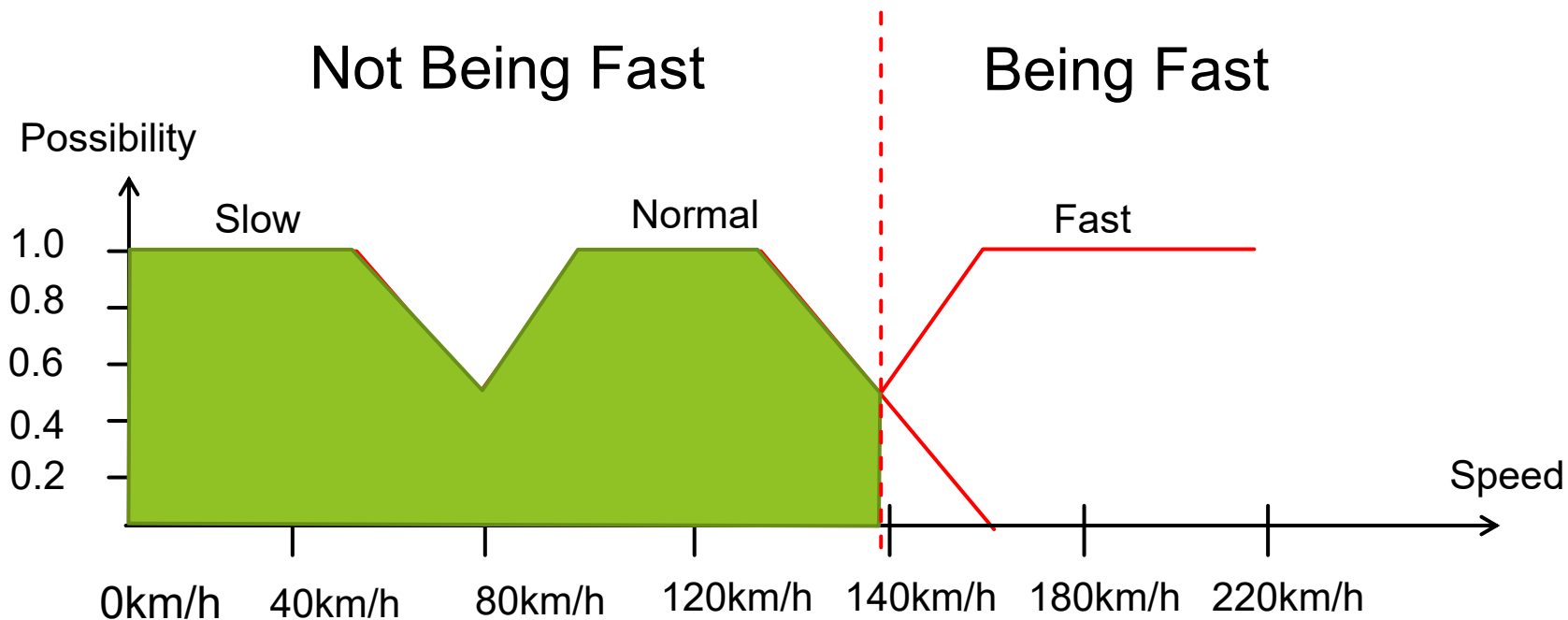
Complement (or Negation) of a Fuzzy Set ...

- ▶ It is the operation of negation with a possibility function within a sensor or a system.

$$\mu_{\neg A}(x) = \neg \mu_A(x)$$

Example

- The negation of fuzzy set associated with belief “fast” is:



NOT and MAX

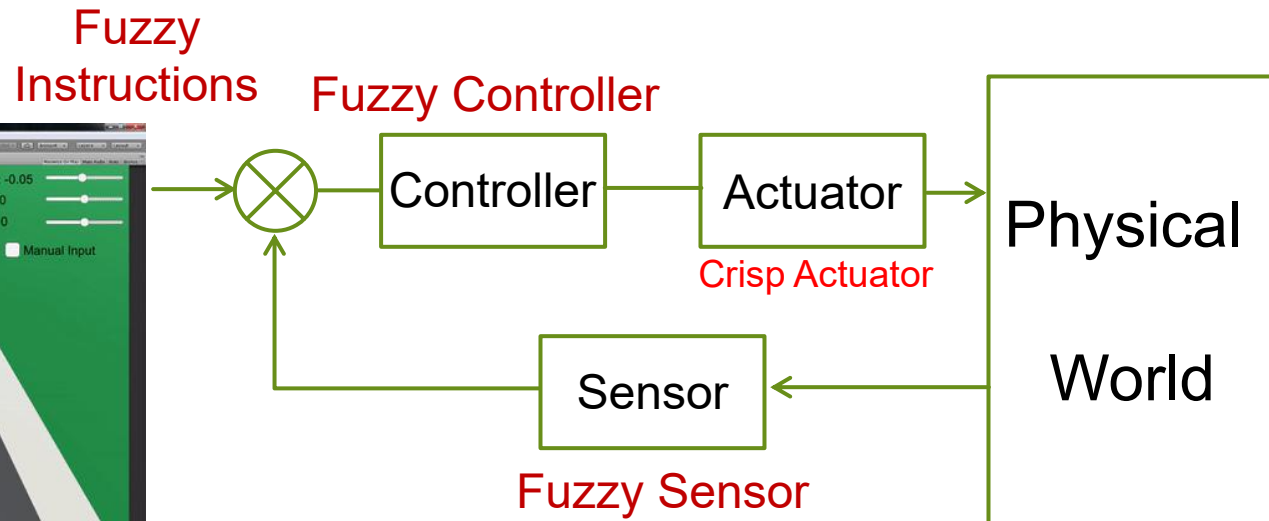
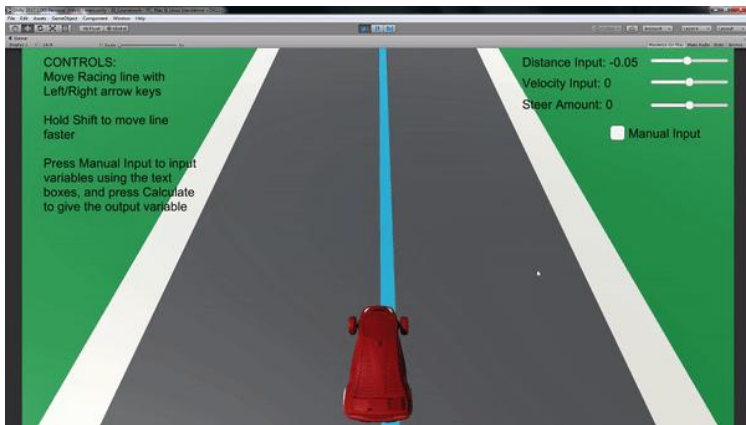
Outline

- ▶ Background Knowledge
- ▶ Fuzziness of Natural Languages
- ▶ Concept of Belief's Fuzzy Sets
- ▶ Concept of Action's Fuzzy Sets

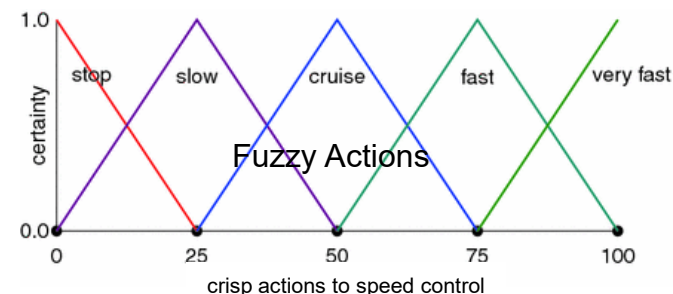
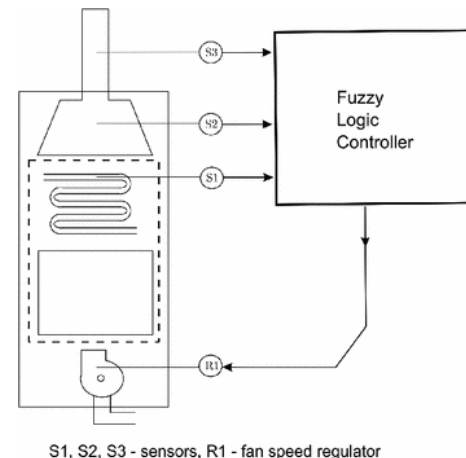
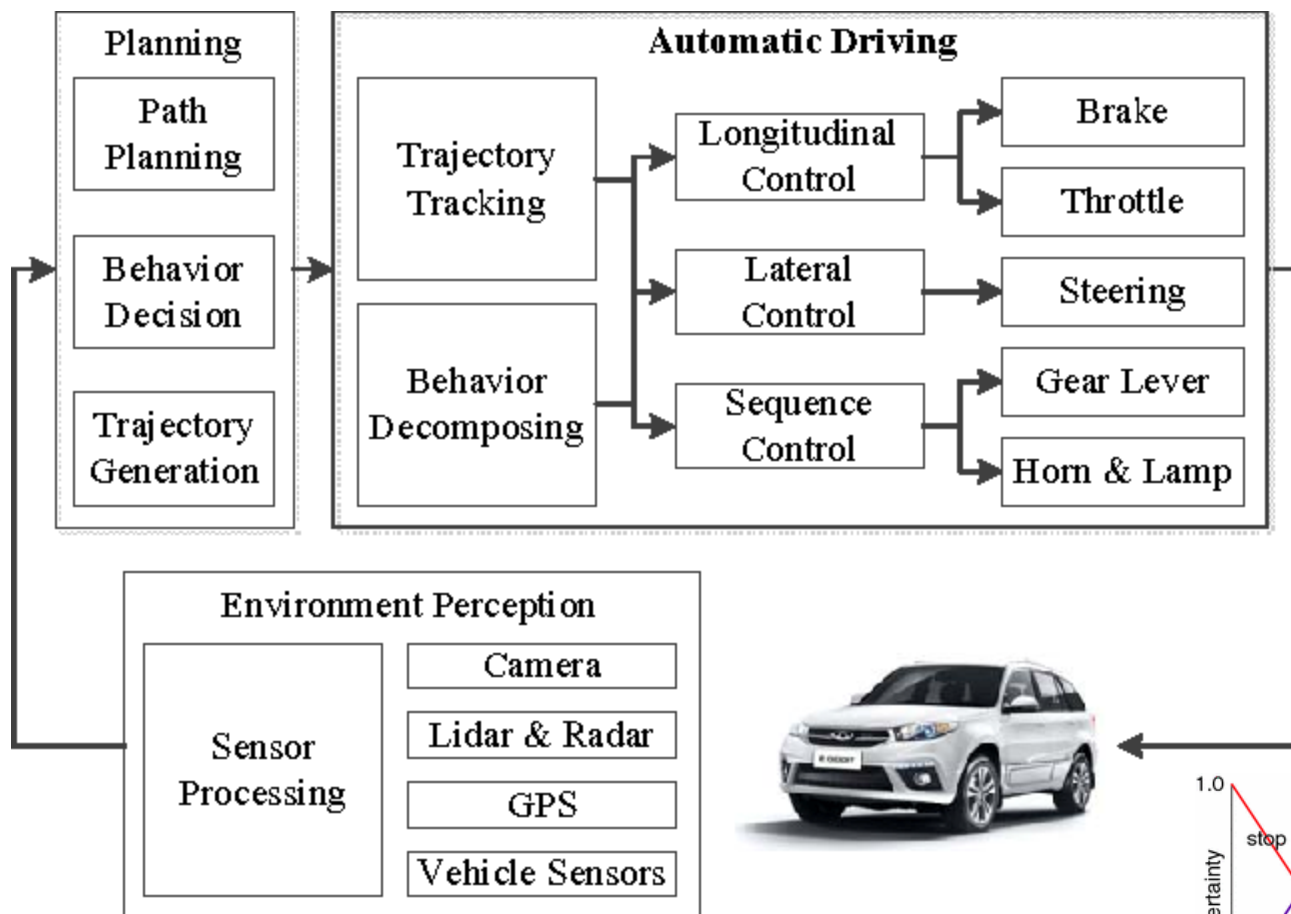
Adaptation to variation of sensory data is an important capability of human intelligence!

Part of Large Knowledge Model

▶ Applications

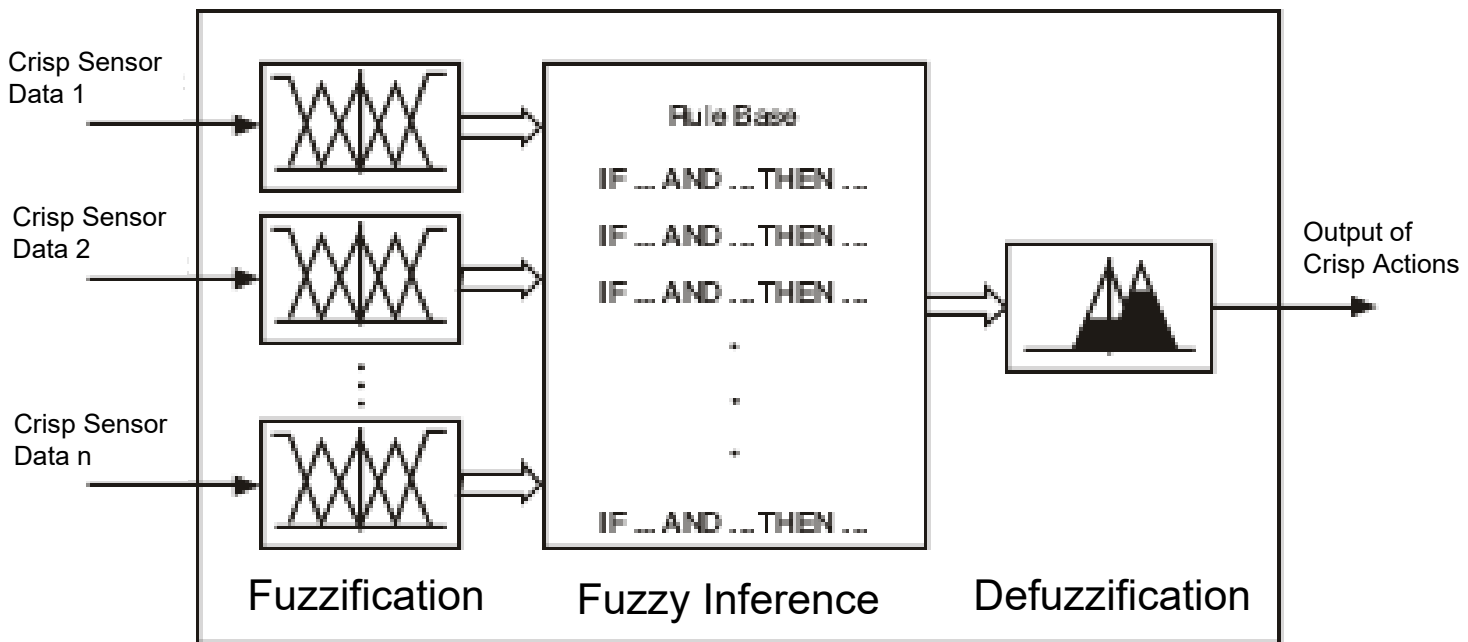


Examples of Implementing Fuzzy Controllers ...



Procedure of Implementing Fuzzy Controllers ...

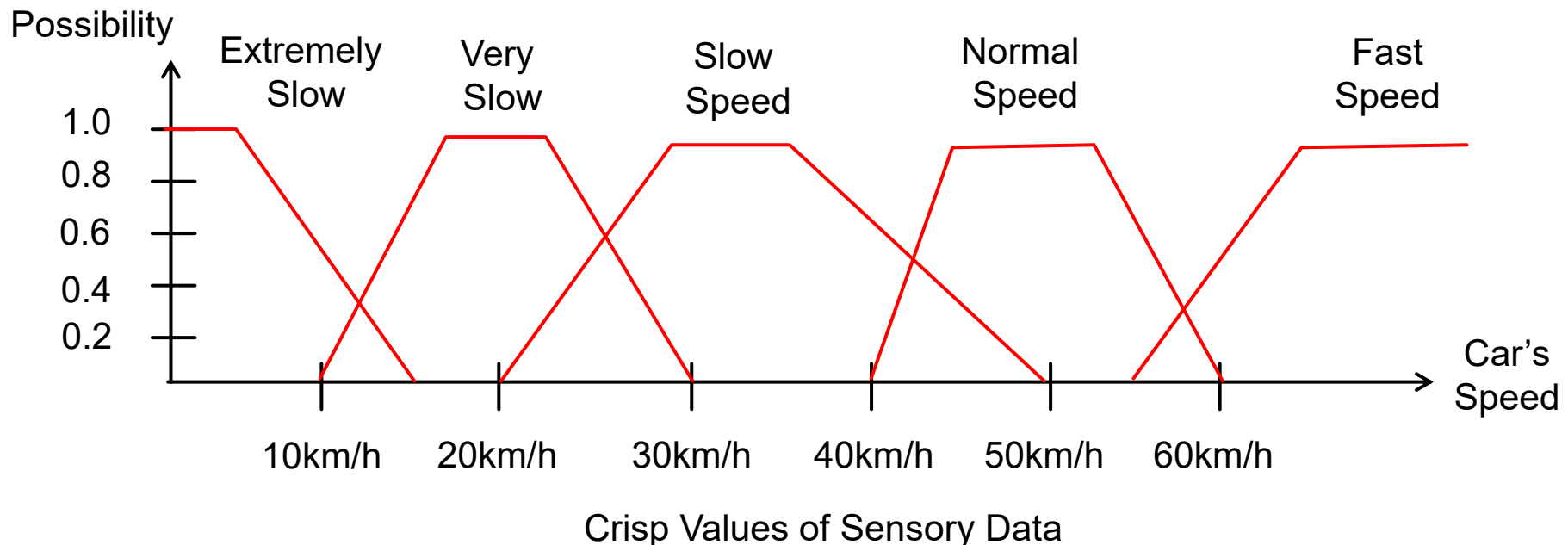
- ▶ Step 1: Transform crisp sensory data into fuzzy conditions (i.e., **beliefs**).
- ▶ Step 2: Transform fuzzy conditions into fuzzy decisions (i.e., also **beliefs**).
- ▶ Step 3: Transform fuzzy decisions into crisp values for action-taking.



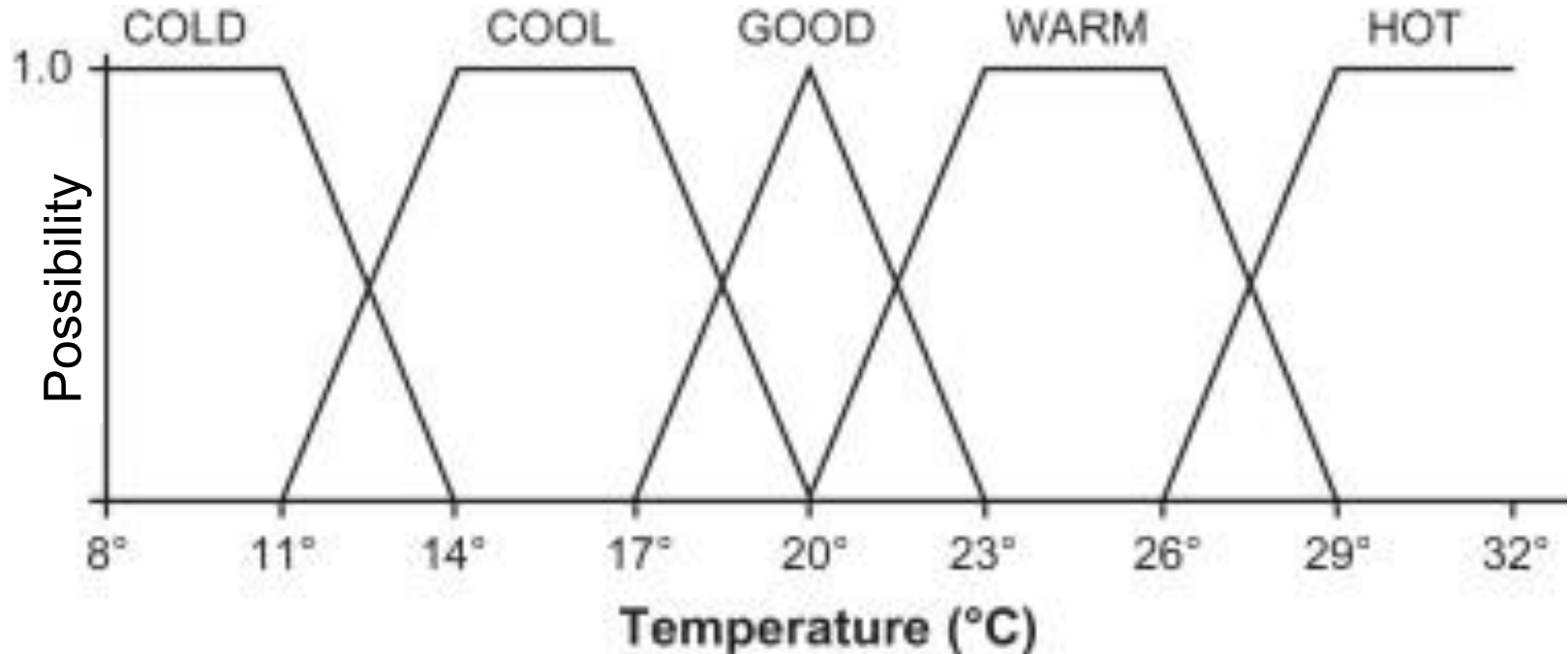
MIMO = Sum of MISO

Step 1: To convert crisp sensory data into fuzzy conditions (i.e., beliefs)

- To use fuzzy belief sets which include possibility functions

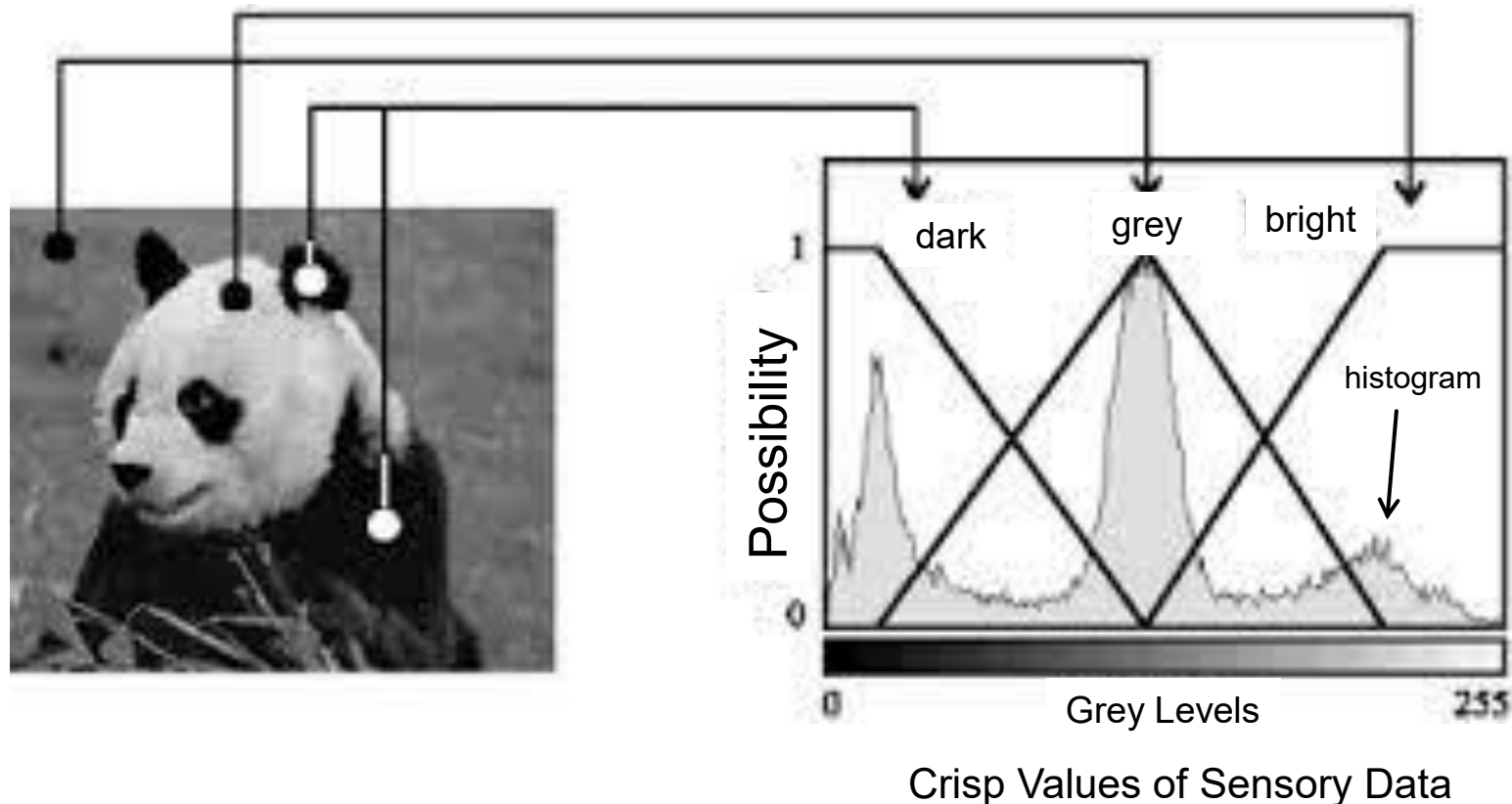


Example to Illustrate Step 1 ...



Crisp Values of Sensory Data

Example to Illustrate Step 1 ...



Step 2: To transform fuzzy conditions into fuzzy decisions of actions

- ▶ To use fuzzy rules such as: if fuzzy conditions, then fuzzy decisions.

For example,

- ▶ If the speed of a car is too fast, strongly increase the braking force.
- ▶ If the temperature in a room is too warm, largely lower the value of temperature setting.
- ▶ If the distance to a frontal vehicle is too far, largely increase the speed of our vehicle.

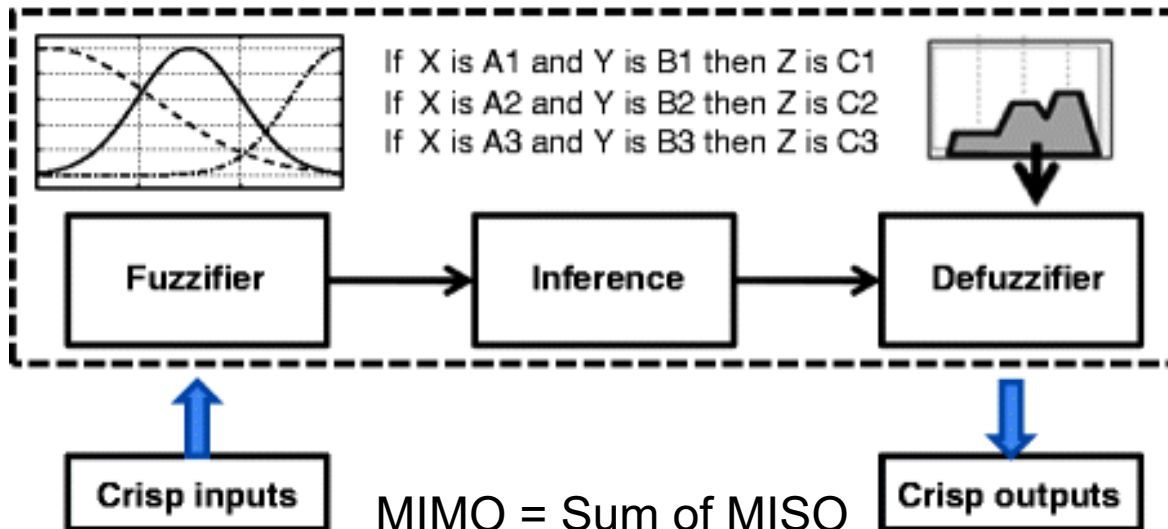
More Examples to Illustrate Step 2 ...

- ▶ If a car is moving too close to the left landmark, then rotate the steering wheel clockwise in a large amount of angle.
- ▶ If a car is following a fast-moving car in front, largely increase the distance to the leading car.
- ▶ If the weather becomes very foggy, largely reduce a car's speed.
- ▶ If error signal becomes too big, largely increase control signal.
- ▶ If the payload becomes too big, largely increase a car's output power.

Step 3: To transform fuzzy decisions into crisp values for action-taking

- ▶ To represent fuzzy decisions of actions.
- ▶ To compute the **weighed sum** of the possibility distributions of all the **permissible** fuzzy decisions of actions.
- ▶ To use the **weighted sum** as the output of crisp value.

Fuzzy Controller



$$V_o = \frac{\sum \mu(a_i)P(a_i)}{\sum P(a_i)}$$

Representation of Fuzzy Decisions of Actions ...

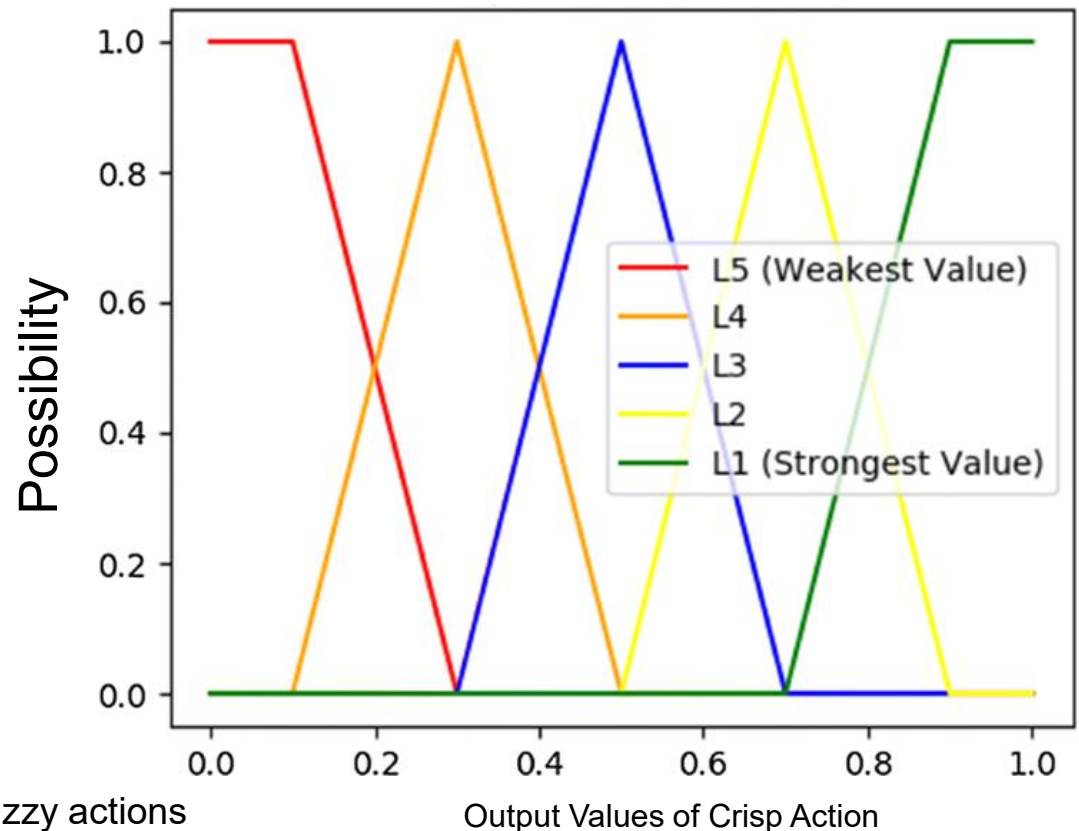
- ▶ To associate a **full-scale** possibility function with each possible decision which is also a fuzzy belief.

Fuzzy Decisions of:

- Action L1
- Action L2
- Action L3
- Action L4
- Action L5

MIMO = Sum of MISO

Fuzzy Decisions of Actions (5 Possible Values)



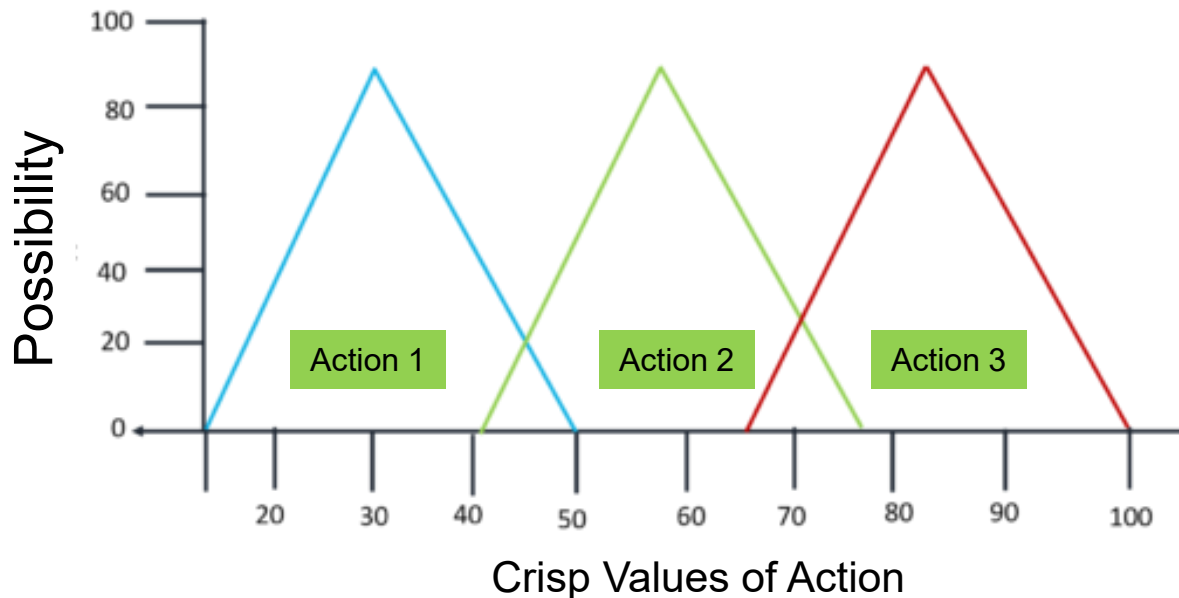
A single sensory data may trigger multiple fuzzy actions

Weighted Sum of Fuzzy Decisions of Actions

$\mu(a_i)$: Mean value of fuzzy decision of action i's possibility distribution.

$P(a_i)$: Possibility of making fuzzy decision of action i.

$$V_o = \frac{\sum \mu(a_i)P(a_i)}{\sum P(a_i)}$$

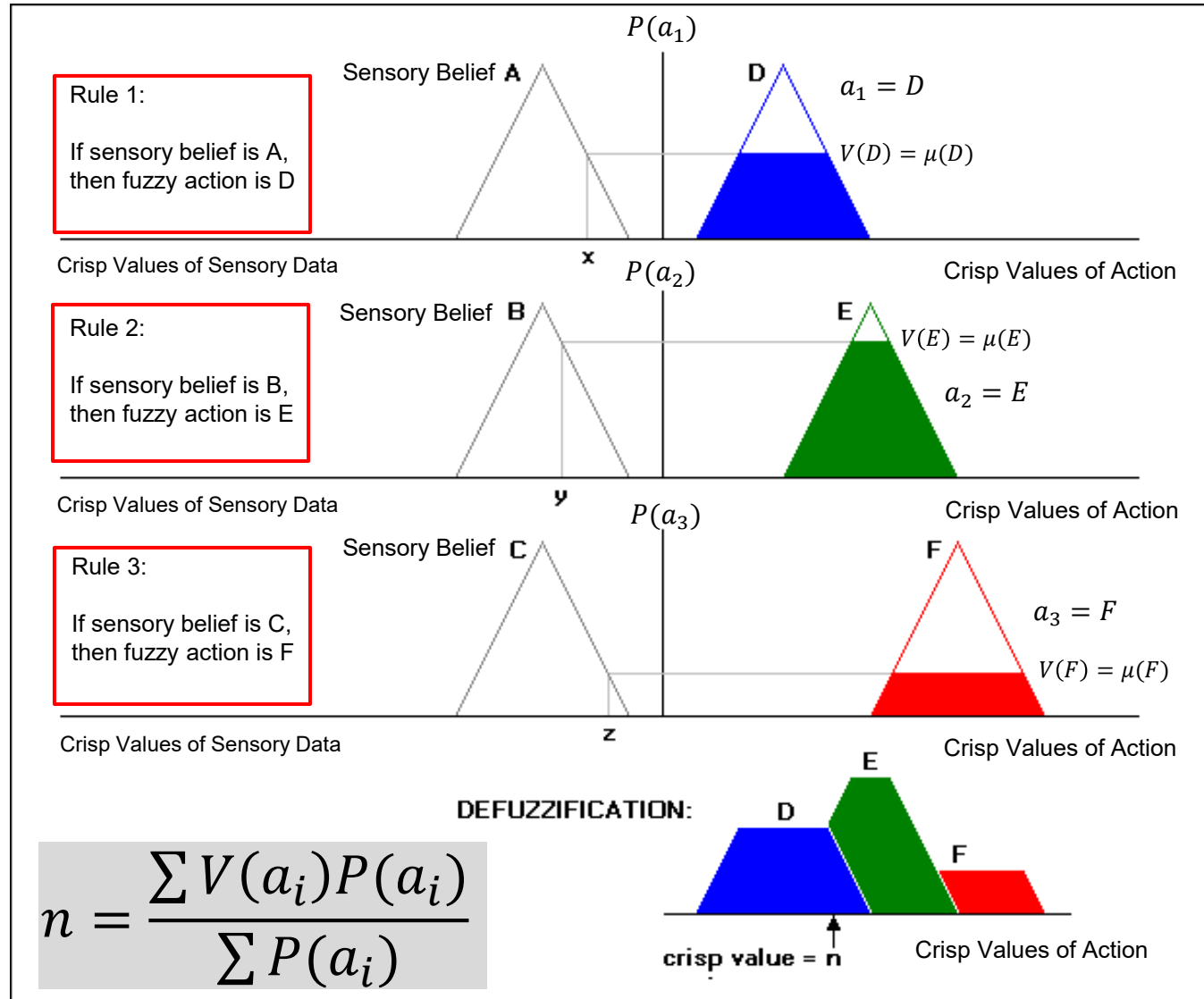


Example of Computing Weighted Sum ...

Sensor Data x triggers action D

Sensor Data y triggers action E

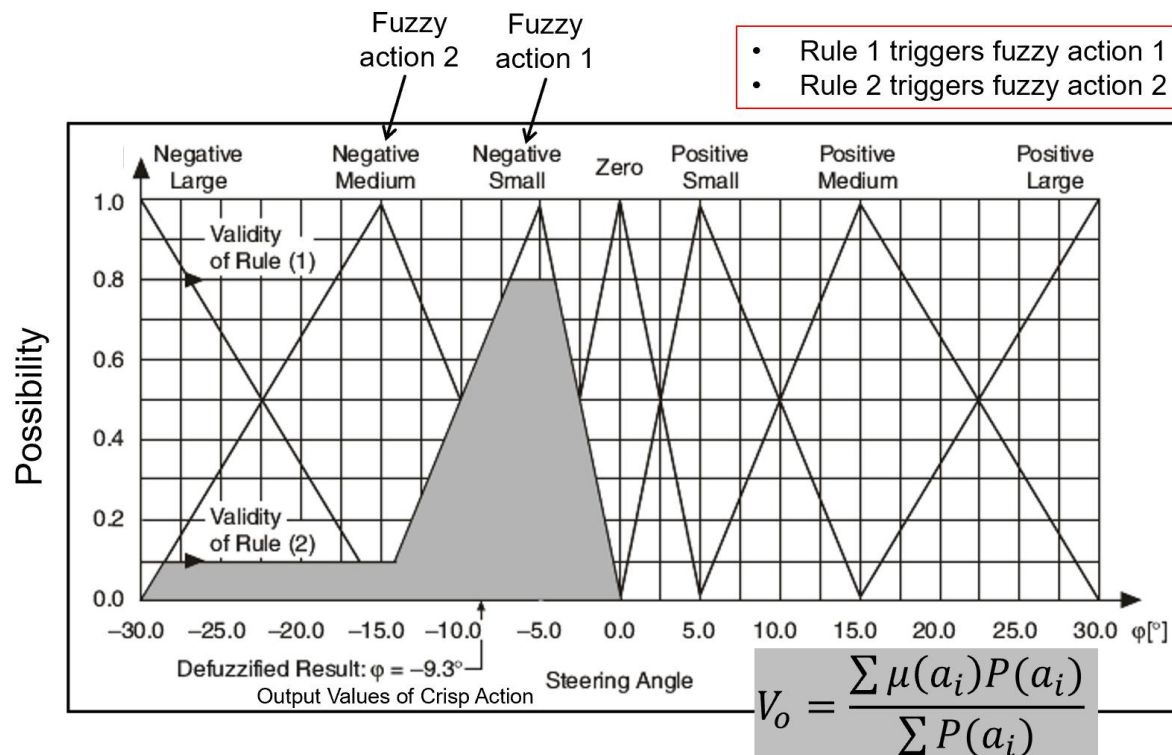
Sensor Data z triggers action F



$$\begin{aligned} \text{Fuzzy Action} &= \\ &= \\ \text{Fuzzy Decision} &+ \\ &+ \\ \text{Crisp Value} & \end{aligned}$$

Another Example of Computing Weighted Sum ...

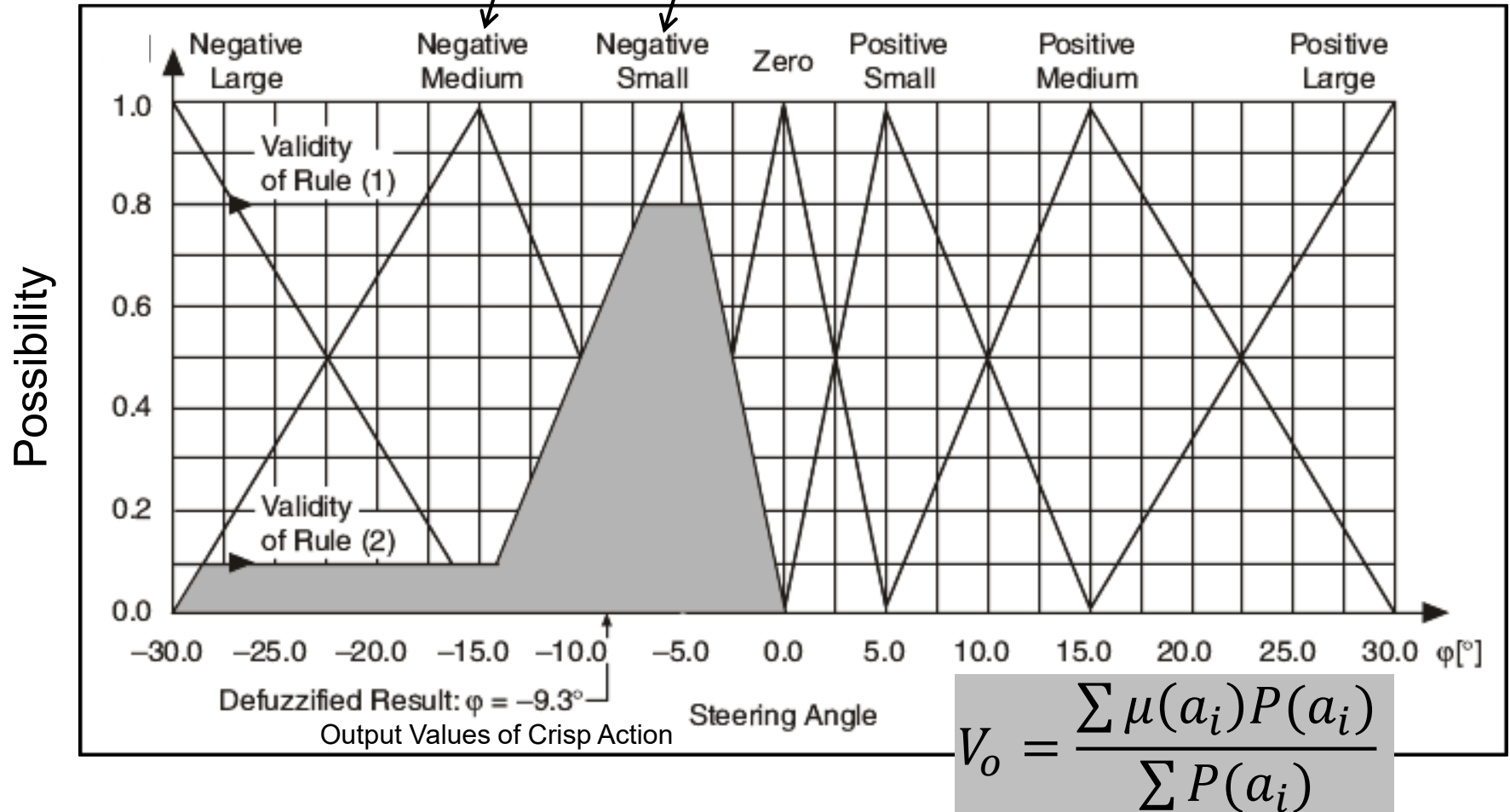
- ▶ Inside a smart car, a fuzzy sensory belief triggers action 1 with the possibility value of 0.8 while a fuzzy sensory belief triggers action 2 with the possibility value of 0.1. If the mean value of fuzzy action 1's possibility distribution is -8.2 degrees and the mean value of fuzzy action 2's possibility distribution is -18.2 degrees, what should be the crisp value of steering angle?



Answer: $V_0 = (-18.2 * 0.1 - 8.2 * 0.8) / 0.9 = -9.31$

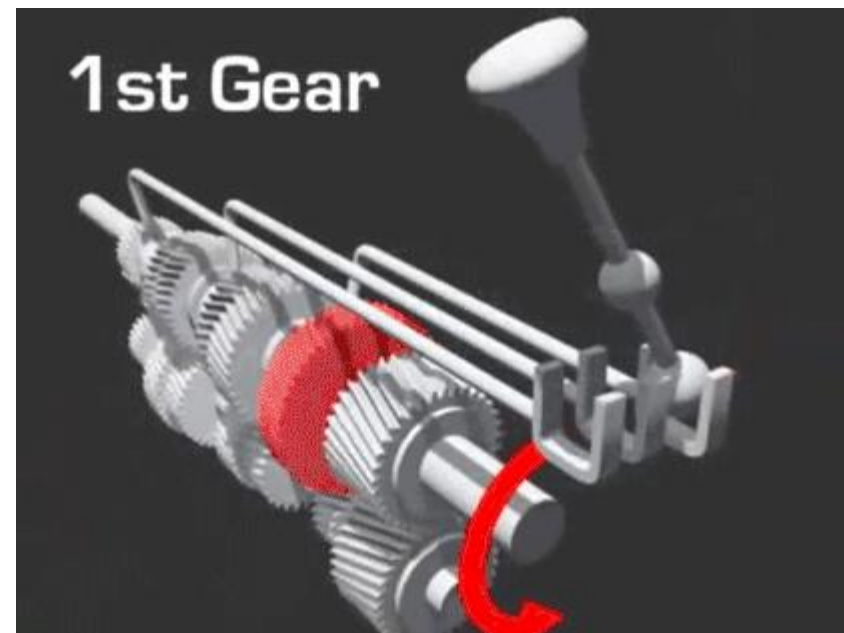
Fuzzy action 2 Fuzzy action 1

- Rule 1 triggers fuzzy action 1
- Rule 2 triggers fuzzy action 2



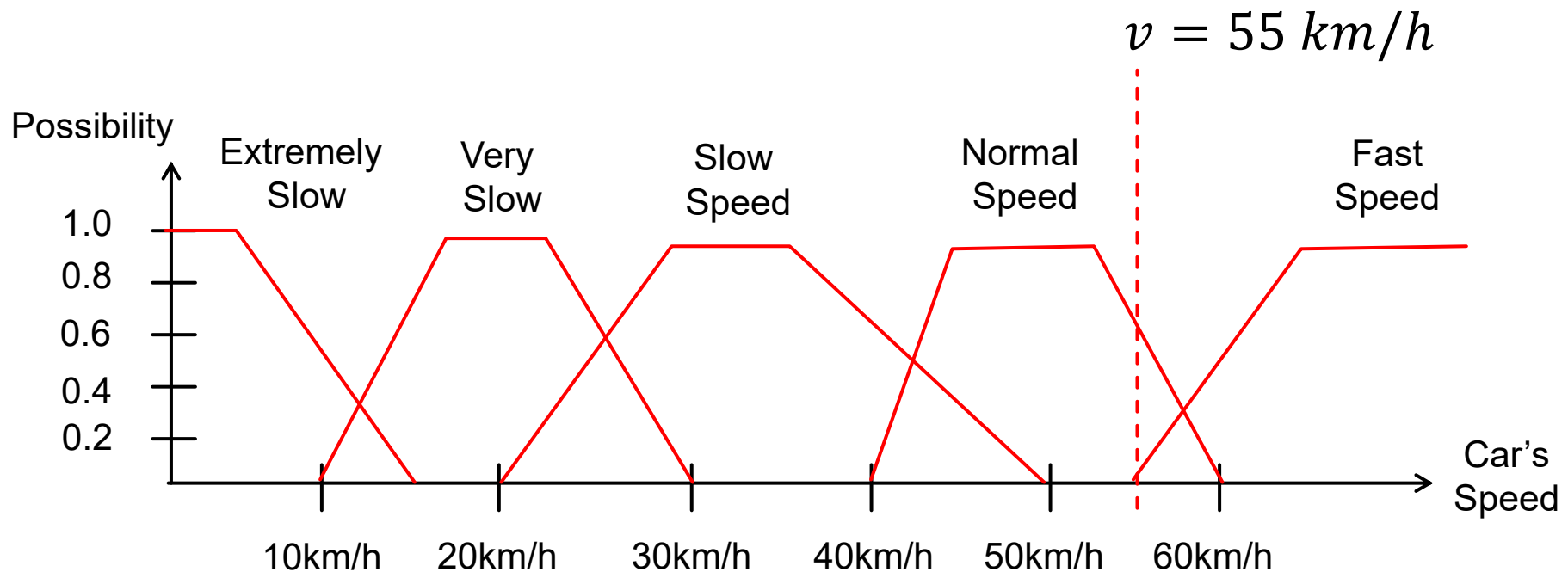
One More Example ...

- ▶ You work in an automobile industry. You are responsible for designing a fuzzy controller for automatic gearboxes with five speeds or gear ratios.
- ▶ What will be your design solution?



Question

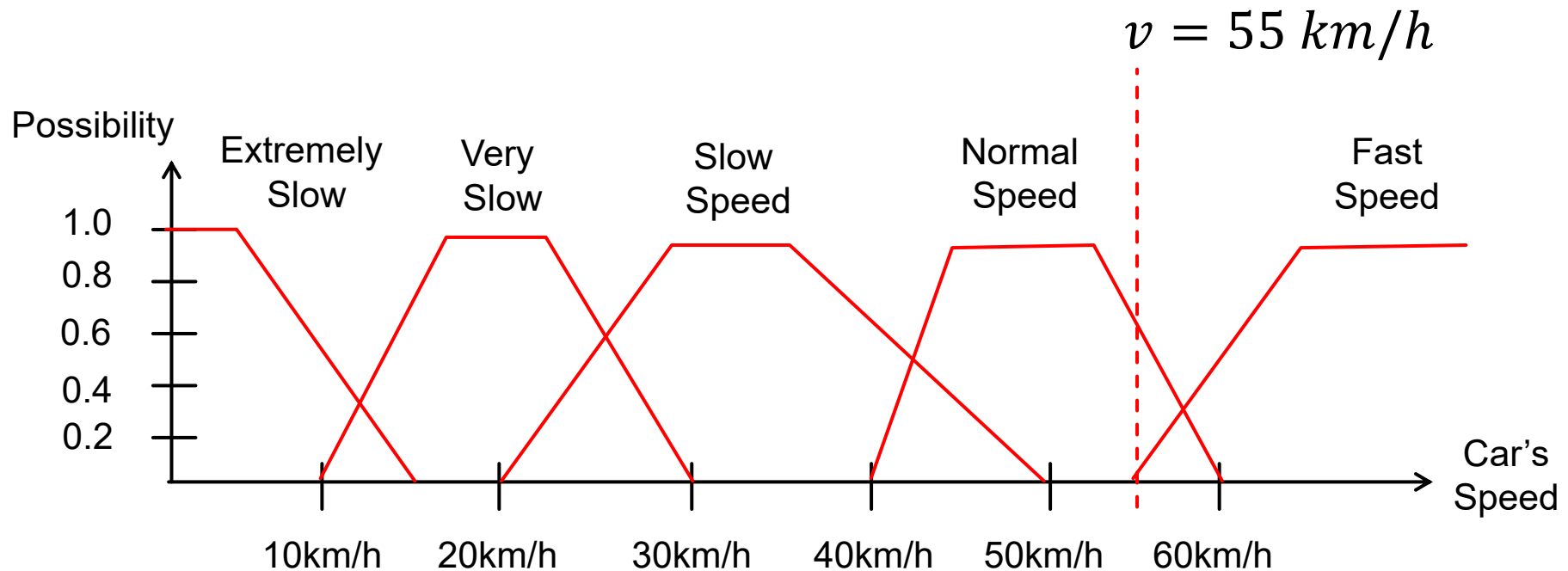
- ▶ If the car's speed is 55 km/h, what should be the gearbox's ratio?



Answer

► Step 1: Do Fuzzification

The sensor's output is 55 km/h. The car is at normal speed.



Crisp Values of Sensory Data

Answer (continued)

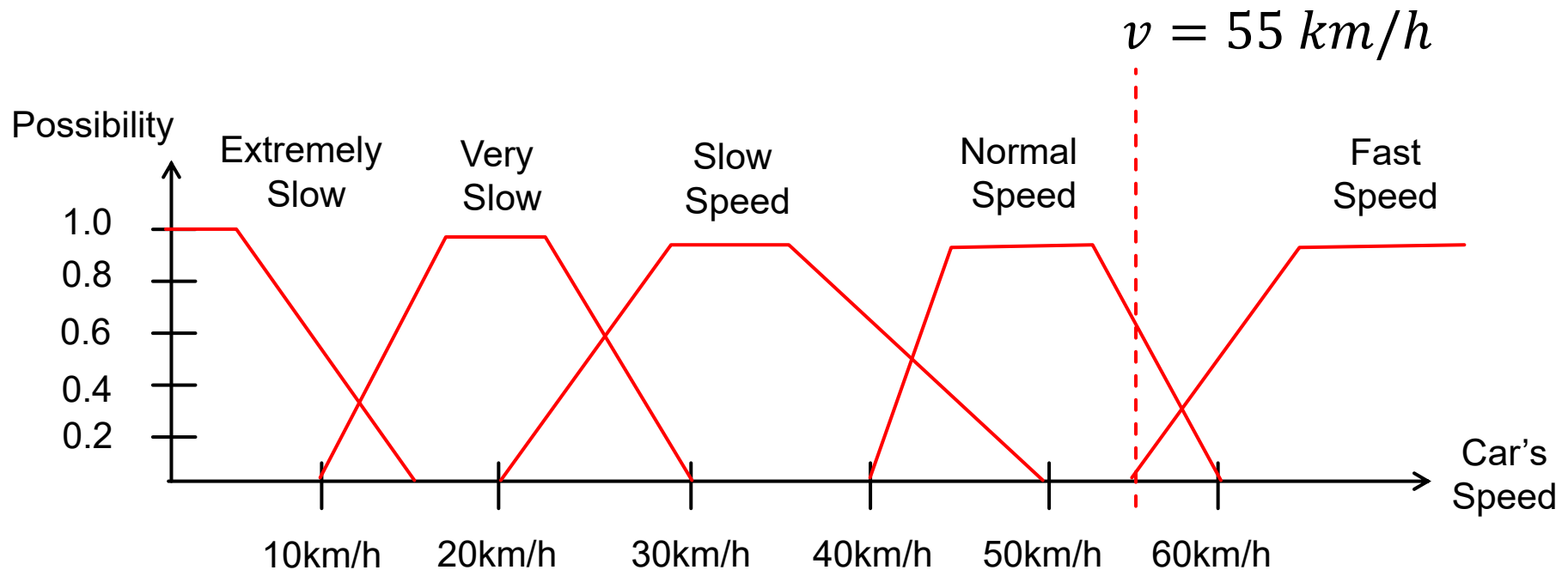
► Step 2: Apply Fuzzy Inference

- If a car's speed is extremely slow, choose speed gear ratio 1.
- If a car's speed is very slow, choose speed gear ratio 2.
- If a car's speed is slow, choose speed gear ratio 3.
- **If a car's speed is normal, choose speed gear ratio 4**
- If a car's speed is fast, choose speed gear ratio 5.

Answer (continued)

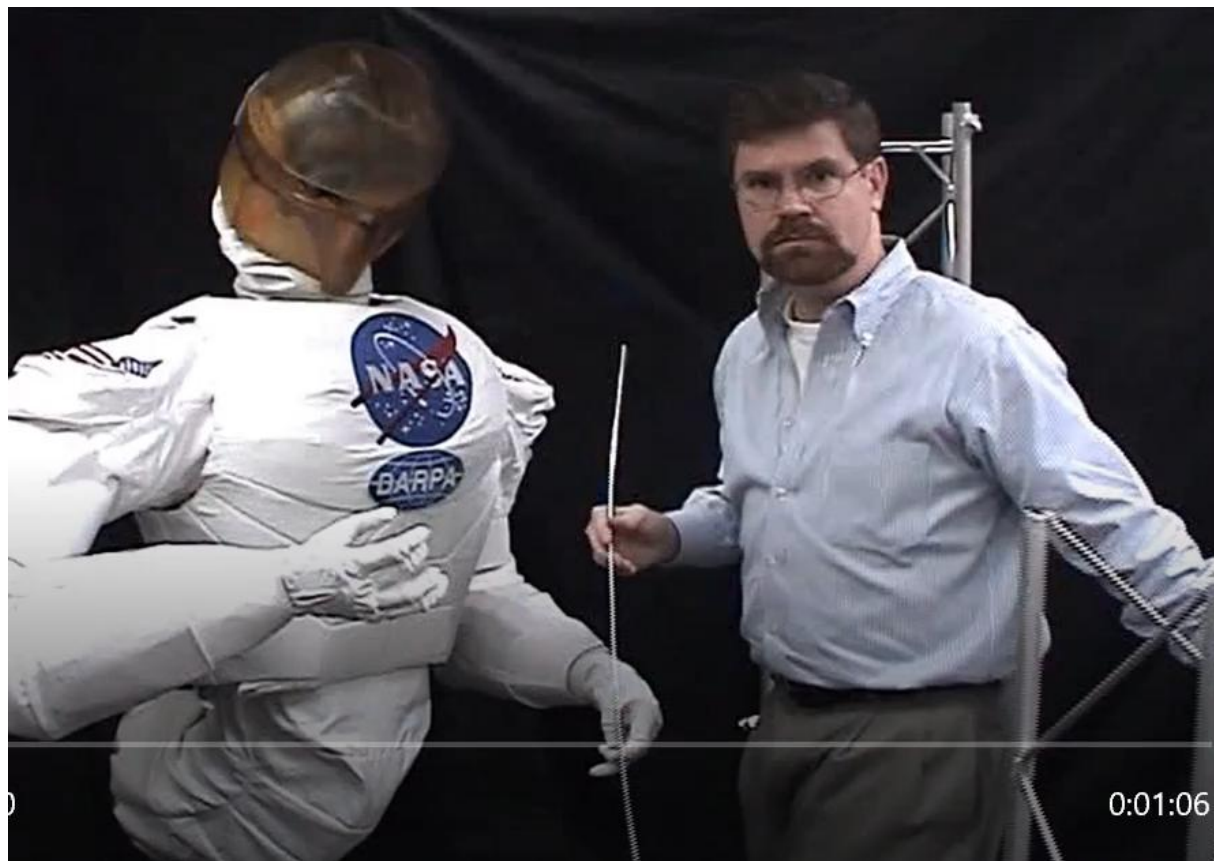
► Step 3: Do Defuzzification

If a car's speed is normal, set speed gearbox ratio to 4.



Crisp Values of Sensory Data

Demonstration of Fuzzy Control with Outer Loop of Fuzzy Perception, Fuzzy Planning and Crisp Control ...



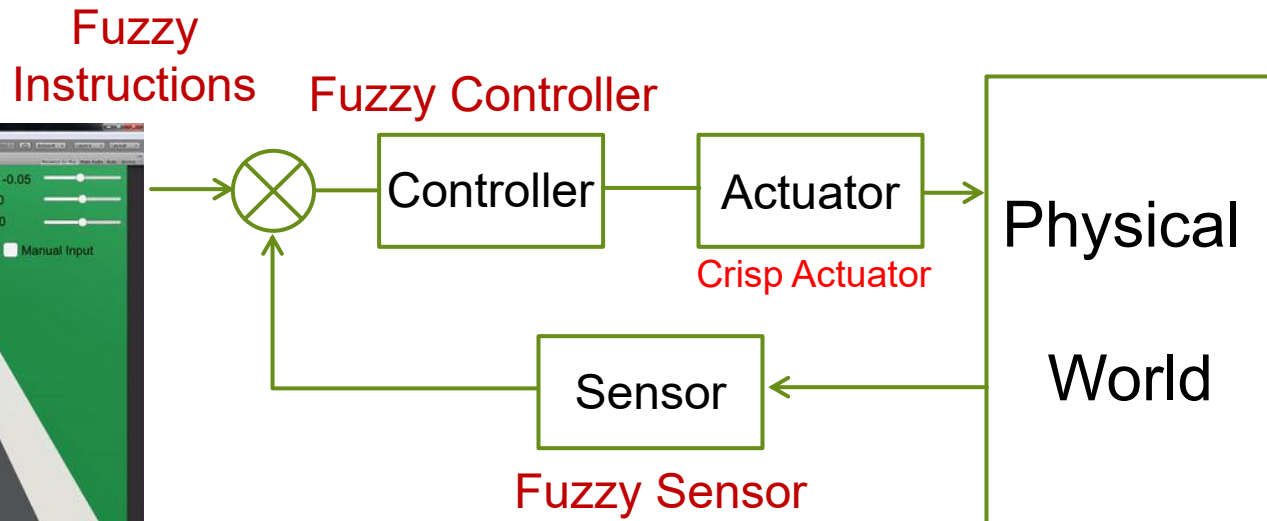
Summary of Lecture 4

- ▶ Background Knowledge
- ▶ Fuzziness of Natural Languages
- ▶ Concept of Belief's Fuzzy Sets
- ▶ Concept of Action's Fuzzy Sets

Adaptation to variation of sensory data is an important capability of human intelligence!

Part of Large Knowledge Model

▶ Applications



Summary of Module 1

▶ Lecture 1:

▶ Basics of Physical World

▶ Lecture 2:

▶ Randomness of Physical World

▶ Lecture 3:

▶ Basics of Conceptual Worlds

▶ Lecture 4:

▶ Fuzziness of Conceptual Worlds





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“Ask not what your country can do for you – ask what you can do for your country,” - John F. Kennedy

“Do not think that you are needy – think that you are needed in the world”, - Manis Friedman

“Study will make you knowledgeable, resourceful, and hence more needed”, - Xie Ming

Thank You for Listening!